Center For Distance and Online Education

Punjabi University, Patiala

Class : B.Ed.- I **Semester : I**
Paper : IV And V (Teaching of Math) **Unit:I**
Medium :English

LESSON No.

1.1 : Nature of Mathematics: Meaning, Nature, importance, values of Mathematics.
1.2 : Axioms, postulate, assumptions, and hypotheses in mathematics.
1.3 : Historical development of notations, Metric System, Logarithms and Computer Mathematics
1.4 : Contribution of Mathematician: Srinivasa, Ramanujan Aryabhata and Bhaskaracharya
1.5 : Euclid and Pythagoras

Department website : www.pbidde.org

PAPER-IV & V: PEDAGOGY OF A SCHOOL SUBJECT (PART-I)

(ix)TEACHING OF MATHEMATICS

SUBJECT CODE: EDUBED1104T
SUBJECT CODE : EDUBED1105

M.M. 50
External: 35
Internal:15

(A) COURSE OUTCOMES

To enable the pupil teacher to:

- Understand and appreciate the uses and significance of Mathematics in daily life.
- Learn various approaches of teaching Mathematics and to use them judiciously.
- Learn the methods of providing instruction for the classroom.
- Organize curricular activities.
- Appreciate activities to develop aesthetics of Mathematics.
- Update their knowledge of content in mathematics.
- Understand the different teaching aids in the Teaching of Mathematics.
- Understand the different techniques for the evaluation of the students of Mathematics
- Evaluate the Student's Performance in Mathematics through the use of the scientific tools.

(B) SYLLABUS

SECTION– A

(i) Nature of Mathematics: Meaning, nature, importance and value of mathematics; Axioms, postulates, assumptions in mathematics;

(ii) Historical development of notations and hypothesis in mathematics; Contribution to mathematics (Ramanujam, Aryabhatta, Bhaskaracharya, Euclid, Pythagoras).

SECTION– B

(iii) Objectives: Aims and objectives of teaching mathematics in elementary and secondary schools; Bloom's taxonomy of educational objectives and writing objectives in behavioural terms.

(iv) Pedagogical Analysis: meaning and need and procedure for continuing pedagogical analysis. Classification of content, objective activity andexperiment, evaluation, etc. Arithmetic (Number systems, Fractions, Ration and proportion, profit and Loss, simple and compound Interest). Algebra (Polynomials, Linear equations, Quadratic equations Arithmetic Progressions), Geometry (Congruent and Similar triangles, Constructions and Circles), Trigonometry (t-ratois, Heights and distances), Statistics (Measures of Central Tendency and Graphical Representation of Data)

Activities (Any one of the following)

(i) Teaching aid from the 3-dimentional aspects
(ii) Creative way of teaching of mathematics at elementarylevel
(iii) Preparing aquestion bank for mathematics.

Head,
Department of Education & C.S.,
Punjabi University, Patiala

(C) BOOKS RECOMMENDED

1. Aggarwal, A.M. (1997). Teaching of Modern Mathematics. New Delhi: Dhanpat Rai Publishing Co.
2. Banga, Chaman Lal (2012). Teaching of Mathematics. New Delhi: Shipra Publications
3. Butler H., Charles, W& Lynwood, F (1951). The Teaching of Secondary School Mathematics. New York: McGraw Hill.
4. Bloom, B.S. (1956). Taxonomy of Educational objectives: the classification of educational goals (is ted.) New York: Longmans Green
5. Chambers, Paul (2010). Teaching mathematics- Developing as a Reflective Secondary Teacher. New Delhi: SAGE
6. Gakhar, S. C. & Singh, Raminder (2005). Teaching of Mathematics, N.M. Publishers
7. Taylor, Helen and Harris, Andrew: Learning and Teaching Mathematics.
8. Hansen, et al: Children's Errors in Mathematics.
9. Witt, Marcus: Primary Mathematics for Trainee Teachers.
10. Chambers, P: Teaching mathematics in the secondary school.
11. Butler and Wren: The Meaning of Secondary School Mathematics
12. Chadha, B.N.: The Teaching of Mathematics
13. Gakhar, S.C. and: Teaching of Mathematics
14. Singh, Raminder
15. Kumar and: Teaching of Mathematics
16. Ratnalikar, D.N.
17. Mangal, S.K.: Teaching of Mathematics
18. N.C.E.R.T. Text Books (6th Class to10th Class)
19. Sidhu, K.S.: The Teaching of Mathematics
20. Travers, et al: Mathematics Teaching

(D) EVALUATION

External Examination	35 Marks
Internal Assessment	15 Marks
Attendance	3
Written Assignment/Project work/Response Sheets	6
Two Mid-term Examinations/House Test	6

(E) INSTRUCTIONS FOR THE PAPER-SETTER

The question paper will consist of three Sections: A, B, and C. Section A and B will have two questions from the respective sections of the syllabus and will carry 12 marks each. Section C will consist of 5 questions of 2 marks each and one objective type question of one mark which will cover the entire syllabus uniformly.

(F) INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt one question each from the sections A and B and the entire section C.

Head,
Department of Education & C.S.,
Punjabi University, Patiala

LESSON NO. 1.1 **AUTHOR : SHRI DARSHAN SINGH**

1.1.1 Objectives
1.1.2 Introduction and Meaning
1.1.3 Educational values of Teaching Mathematics
1.1.4 Place of Mathematics in Daily life
1.1.5 Summary
1.1.6 Self Check Exercise
1.1.7 Suggested Questions
1.1.8 Suggested Books

1.1.1 Objectives

After reading the lesson you will be able to:

1. Define Mathematics
2. Explain Educational values of Teaching mathmathics
3. Explain place of mathemathics in Daily life
4. Apply values in day to day life.

1.1.2 Introduction and Meaning

It is a matter of common observation that people know much about Mathematics yet they may be handicapped about its basic nature. They may know and understand the typical problems and theorems of Mathematics yet may be ignorant about its fundamentals. So to start with, it will be wise to start our discussion with the definition of Mathematics and its basic nature.

What is Mathematics?

Mathematics has been defined in many ways. Few important definitions may serve our purpose to acquaint us clearly with the meaning of Mathematics. According to one view Mathematics is defined as "the science of quantity and space."

According to one dictionary, "Mathematics is science of number and space." According to another dictionary, "Mathematics is the science of size and number." (of which Arithmetic, Algebra, Geometry and Trigonometry are branches). Yet another dictionary defines it as "the science of measurement, quantity and magnitude."

So we can say Mathematics is the logical study of shape, arrangement, quantity and many related concepts.

From the above few definitions, it is clear that mathematics is a systematised, organised and exact branch of Science. It is a Science which deals with

QUANTITATIVE aspects of our life and knowledge. It is a type of Science that helps us in solving problems of life involving numerations and calculations. It provides a special type of training to our mind which is very much helpful in drawing necessary conclusions and interpreting various ideas with useful meaning.
It has its own language, tools and mode of operations. Its language tools and operations are of tremendous importance to understand and derive useful relations for the nature's work and complicated problems of life. Interpretation of social and natural phenomena by mathematics is the basic for modern civilization or in other words, we may say Mathematics is the basis for development in Science and Technology which in turn controls everything.

1.1.3 Educational Values of Teaching Mathematics

There are few questions which confronts our mind many times. So to discuss about educational values in detail, it will be good to write these questions here for review.

- — Why everybody needs to study Mathematics ?
- — Why educationists are eager to teach this subject to every body ?
- — What is the use of putting too much time and energy to the teaching and learning of this subject ?
- — How Mathematics is useful subject ?
- — Does it help one to learn other subjects in any way ? and so on.

The answers to all the above questions lies in the educational value of this subject. This subject is very important from many angles. This can be seen from the educational values of this subject. These values may be classified broadly under the following headings :

1. Utilitarian or Practical Value,
2. Disciplinary Value,
3. Cultural Value,
4. Social Value,
5. Moral Value,
6. Aesthetic or artistic Value,
7. Intellectual Value, and
8. International Value.

Now let us discuss briefly about these values :

1. Utilitarian or Practical Value :

Perhaps it is the only value from which Mathematics is considered as a useful subject. For a common man, Mathematics is a subject which has an importance in day to day life. Everybody irrespective of his profession, needs some amount of Mathematics to live a balanced life. Even a labourer has to count his wages and want to spend it in such a way that he should meet both ends. Similarly a farmer, a house wife, a hawker etc. has to calculate their income and expenditure. So it is clear that Mathematics is not only important for educationists, statisticians, bankers, administrators, finance minister, rather is necessary for everybody.

We can see practical value of Mathematics from another angle too. As we generally say that modern civilization is due to the advancement in Science and Technology. Yet many of us are not aware of the bases of these science's and technologies. If we study deeply then realize that the development in these sciences and technologies is due to the advancement in the field of Mathematics. By comprehensive study it is found that development in science and Technology is directly related to the advancement in higher Mathematics. In the words of Mellor, J.W., "It is almost impossible to follow the later developments of Physical and general Chemistry without a working knowledge of higher Mathematics."

The use of Mathematics is not only in the field of physical and bio-sciences rather Mathematics is now widely applied to social sciences also. It has direct or indirect relationship with almost all the subjects. That is why it is termed as "the science of all sciences and the art of all arts."

2. Disciplinary Value :

In the words of Locke : "Mathematics is a way to settle in the mind a habit of reasoning". If we study this statement then it is understood that this subject trains one's mind in a way which is necessary to solve problems both in the field of life and education in a scientific and systematic way. Mathematics disciplines learner's mind. By the continuous study of Mathematics one can develop mathematical reasoning which is very important to solve all types of problems in any sphere, one may face.

Reasoning in Mathematics possesses few unique characteristics mentioned below :

(i) Characteristics of Originality :

It is a matter of common observation that the most work of Mathematics demand original thinking. There is no place for duplicacy, and cramming, rather the study in this subject requires original thoughts. Now it is not considered as a subject of chalk and black board only, rather it is a subject of new innovations and new techniques and the development of these require only original thinking and not repetitions.

(ii) Characteristics of Accuracy :

In Mathematics there is no place for wrong and false interpretations and observations. It needs very perfect, dynamic, systematic and scientific approach to discuss the authentic ideas. Mathematics demands accuracy and exactness. There is no two opinions about any fact, principles and thought of Mathematics.

(iii) Characteristics of Simplicity :

Reasoning in Mathematics resembles to the reasoning of life. When we face a problem in life we collect data about the problems, analyse and data and then by reasoning we come to the conclusion. Similarly in Mathematics we also proceed in the same way. Hence the reasoning used to solve the Mathematics problems is of simple type. It also teaches that definite facts are always expressed in a simple language. So if you want to be understand, you must express yourself in a definite or simple way.

(iv) Characteristics of certainty of results :

There is no place for subjectivity in mathematics. Only objective thinking is encouraged and accepted. No two persons differ at any concept, idea, principle etc. of mathematics. The student can also verify their results by their own. They can detect their weaknesses and good points at any time and also by doing so they can make self efforts to remove their discrepancies. Study of mathematics makes themselves conscious about every sphere of life. It regulates their activities in such a way that leads to become a perfect man.

(v) Characteristic of Similarity to the reasoning of life :

Clear and exact thinking is as important in daily life as in mathematical study. Before starting with the solution of a problem, the student has to grasp the whole meaning. Similarly in daily life, while undertaking a task, one must have a firm grip on the situation. This habit of thinking will get transferred to the problems of daily life.

(vi) Characteristic of verification of results :

Results can be easily verified. As already pointed out, this gives a sense of achievement, confidence and pleasure. This verification of results is also likely to inculcate the habit of self criticism and self evaluation. After making any attempt in life, the child would like to satisfy himself about its success or failure.

It may further be noted that acquisition of knowledge is not only the purpose for which a child is sent to school. Knowledge often becomes out of date, as these are days of rapid progress and quick changes. Therefore in this ever advancing society, the important thing is not only to learn facts, but also to know how to learn facts. The main thing is not the acquisition of knowledge but the acquirement of the power of acquiring knowledge. This is the discipline of mind, which everybody should aspire for.

Secondly knowledge itself becomes real and useful only when mind is able to apply it to new situations. In mathematics there is again a vast scope for application. Ability to apply knowledge to new situations is inculcated in students.

Thus we have seen that mathematics is a unique subject which possesses such characteristic of reasoning. Thus mathematical reasoning is responsible for disciplining one's mind.

3. Cultural Value :

By culture of a society or nation we mean the mode of living of its inhabitants. As we take the example of Science we all know that scientific discoveries are responsible for bringing changes in the mode of living. And it is also known to us that Mathematics is the basis for all sciences. Modern civilization is the product of development in Science and Technology, which in turn depends upon development in Mathematics.

In case of some developed civilization of the past such as Babylonian, Greek and Indus valley, we found that the Mathematics of these civilizations was very much developed. But there are some tribes known today which are very much uncivilized and their development in Mathematics is almost nil. Similarly, if our modern civilization is so developed, it is only due to the higher development of Mathematics. So from above discussion, it can easily be concluded that the development in civilization is directly proportionate to the development Mathematics of that civilization. By knowing the development in mathematics of any society, country or nation, we at once judge the standard and mode of living of the people of that very society, country and nation. For instance, it may not be altogether a matter of chance the Greeks, the greater geometers of the age, were also the most successful in fine arts.

4. Social value :

What we are enjoying in our social set up and planning to get in future is based on the study and knowledge of Mathematics. In one way or the other, directly or indirectly Mathematics influences our way of life. Napoleon once said, "The progress and improvement of Mathematics are linked to the prosperity of the state."

It is clearly understood from the above statement that research and development of Mathematics reconstruct social and cultural values and hence our mode of living. Thus our social development is linked to that of Mathematics.

5. Moral value :

By the regular study of Mathematics one not only germinate but also nourish character formation and moral development. It helps to develop habits of good character like truthfulness, honesty, regularity, justice, duty boundness, patience, confession, respect for other's opinion, discrimination between good and bad, expression of thoughts in a systematic and scientific process. It generates the concept of morality in its learners.

Mathematics helps in developing proper moral attitude. There is no place for prejudiced feelings, biased attitudes, doubts and half truths in the solution of problems in Mathematics. In this way Mathematics is the only subject which helps in impersonal analysis, correct reasoning and valid conclusions.

6. Artistic or Aesthetic Value :

Those who did not get proper opportunities to study mathematics have developed a wrong notion in their minds that mathematics is dry and uninteresting subject. For lovers of mathematics there is all beauty, art, music and fineness in the subject. One finds a huge treasure of pleasure after getting success in the solution of a mathematics problem.

There is no exaggeration in saying that mathematics is the creator as well as nourisher and saviour of all arts. What we enjoy in the arts like Drawing, Painting, Architecture, Music or Dance etc. is all due to mathematics. Mathematics regularity, symmetry, order and arrangement play a leading part in beautifying and organising the work of these arts. Mathematics does not only give pleasure through its application to various arts but also entertains with its own riddles, games and puzzles.

7. Intellectual value :

Mathematics is a useful subject from intellectual development point of view. Regular study in this subject helps to develop powers of imagination, memorization, observation, invention, creativity, logical thinking and systematic reasoning among the learners. One develops power to acquire knowledge rather to collect information. In this ever advancing society, knowledge taken today may be out of date tomorrow. So to acquire knowledge is not our ultimate aim rather power to acquire knowledge should be the aim. Moreover the knowledge is useful only when we know how to use or apply it in solving our problems. Therefore, both the things–the power of acquiring knowledge and skill to use the acquired knowledge properly at the hour of need are only aimed through the teaching of mathematics. Mathematics helps to acquire power which is necessary and basic for dynamic thinking. One can adjust to changed society in a better way, if one is well conversant with basic laws of Mathematics. Mathematics is only authentic discipline which gives us a full knowledge about life, because this is a system which can scientifically operate and observe any problem. It is based not on theoretical concept rather it is completely concerned with practical considerations.

8. International Value :

Mathematics helps in creating international understanding and brotherhood among different nations, cultures, societies and countries. Development in any branch of Mathematics cannot be restricted only to one nation or country. The man made boundaries are not barriers for exchanging latest development in Mathematics. Any new idea or thought developed in any corner of the world is popularized soon throughout the world. Countries generally exchange latest development in higher Mathematics. So each country is very close to each other in the latest development in this subject. In other words Mathematics creates a link among different nation or societies. For instance the difference between underdeveloped countries and developed countries irrespective to their scientific and technological knowledge is rapidly diminishing. As Mathematics is the backbone of Science and Technology hence we can say that it is the Mathematics which is responsible for this international understanding.

Conclusion :

Form the above discussion, it is clear that mathematics is a very useful subject. It has tremendous practical use both in daily life and in the study of other subjects. It disciplines one's mind in such a way that one acquires power rather than more knowledge. Mathematics is closely related to the culture society. We may say as the Mathematics so is the culture. It trains and develops learner's intellectual faculties to the maximum. It also lays foundation for suitable character formation. It bounds nations by developing feeling of international understanding.

1.1.4 Place of Mathematics in Daily Life

Imagine for a moment if we are deprived of Mathematics all together then what will happen ?

Everybody has to calculate his income and plan his family budget to run a balanced life. Though the amount of knowledge in Mathematics needed for a common man is very less and that too of compotation level, yet absolute ignorance about Mathematics is not possible to think. From morning to evening we, irrespective of our profession use some amount of mathematics in the form of its rules, principles and formulae. Even a house wife has to plan her programmes according to her resources. She has to plan for family budget and estimates, writing various expenses etc. So to run a home in a better way she needs the help of Mathematics.

First of all in the primitive tribes when no Mathematics had been developed, people were interested to answer question like, How much ? How many ? How long ? etc. they felt need to answer these questions. The answers to these questions were obtained by the invention of theory of numbers or arithmetic. When the society progressed, Mathematics also developed and new branches emerged like Algebra, Geometry, Trigonometry, Calculations etc. Hence it is clear from the development of Mathematics that it emerged from the felt need of the society and develops.

Science and technologies are consider as the controlling factors of modern civilization. Also it is an accepted fact that Mathematics is the backbone of all Science and technologies. Thus Mathematics can be considered as the controlling factor of modern civilization. It is the modern Technology and Science that in turn control our society.

Most of the occupations are directly or indirectly based upon Mathematics e.g. occupations such as, shop keeping, tailoring, carpentry, taxation, insurance etc. make use of Mathematics directly or indirectly.

So to include with we may say that Mathematics is such a useful subject that regulates our day to day functioning. It may be considered second tongue of a person as it has its own language, tools processes and operation. It is an important instrument to value problems in daily life in a more rational way. The inductive reasoning developed by it is of greatest practical help in our day to day life.

Place of Mathematics in School Curriculum

The main aim of education is all round development of the learner. Every subject gives its due share to attain this ultimate aim. Those subjects get preference in the list of subjects to be taught which has more educational values. In other words every subject is taught due to its educational values. Mathematics is a subject which has educational value. To acquire its basic knowledge is a necessity of each individual. Without its basic knowledge one feels handicapped at all occasions of life. Thus to impart education, at certain minimum levels, in this subject is necessary. This is why we are providing

Mathematics as a subject up to middle or high school standard. Our main aim to impart instructions in Mathematics up to this minimum standard is to make the students efficient to solve mathematical problems of every day life and also enable them to study higher Mathematics. It is a debatable issue that whether Mathematics should be made compulsory up to high school classes or made it optional after middle classes. We will discuss it in the following paragraphs :

Whether Mathematics should be made compulsory up to high classes or made optional after middle classes :

We are in favour of the teaching of the Mathematics as a subject up to some minimum standard as it has many educational values. But there is no one opinion on the topic that up to which class it should be made compulsory. The minimum standard has a great practical implication because we have to safeguard the interests of every student leaving the school after certain minimum level or the student going for higher study. There are generally two view points regarding this topic as discussed below :

Arguments against making Mathematics a compulsory subject up to high classes :

1. Mathematics is a difficult subject. It is clear from the low pass percentage in this subject in comparison to other subjects. This may be due to lack of specific ability and intelligence required. Mathematics is not everybody's cup of tea. So why to make it compulsory for everybody to study it up to high school classes. The amount of knowledge of mathematics needs for a person other than those who want to study higher Mathematics is sufficient up to middle classes. Hence Mathematics after middle classes should be made optional.

2. Everybody has not to get employment in the professions such as engineering, accountancy, banking, surveying, statistics etc. Therefore there is no need to teach Mathematics up to high school stage to everybody.

3. Psychologically too, it is not good to make Mathematics compulsory for everybody up to high school classes. As by doing so we are ignoring the potentialities, capabilities capacities, interest, aptitudes etc. of the students. Student can loose interest in study all together if we compel him to study Mathematics. Hence it is advocated that Mathematics should be made optional after middle classes.

4. Most of the developed countries of the world has made Mathematics a compulsory subject only up to middle classes. Then why it has been compulsory in our country up to metric standard ?

Argument in favour of making Mathematics subject up to high school classes :

1. Mathematics is a very interesting subject. Those who study Mathematics, from not only examination point of view realize that it is very simple subject. The symmetry, similarity involved in the results of theory of numbers are the true examples of its beauty. Actually Mathematics is not a difficult subject rather teaching of Mathematics make it difficult. Our teachers are using traditional methods for teaching this subject. They are not fully aware of the progressive, scientific and psychological methods of teaching. Curriculum in Mathematics is also defective. Good and relevant audio-visual aids are not used by teachers while teaching this subject. Lack of motivation both on the part of the teacher and the students also contribute towards making it a dull and dry subject. Hence Mathematics is not a difficult subject rather all the above mentioned factors contribute to make it so.

2. It is very difficult for a student of middle standard to decide which profession, he will adopt after completion of his studies. It has been seen in some cases that some student show less interest in Mathematics in lower classes. But their interest increases in higher classes. Thus only on this consideration it will not be wise to restrict the scope of students of middle class for some professions like engineering, accountancy, banking, surveying, statistics etc. Let them be wise for two more years to decide about their future.

3. It is now an accepted fact that mathematical ability has close relation with general intelligence. It is not something special as had been considered before. A person who is intelligent in other subjects will be so in Mathematics too. If we teach students with the help of progressive and psychological methods of teaching then it suits to varies interest. Thus teaching of Mathematics up to high classes should be made compulsory.

4. The view that Mathematics is optional after middle classes in some advanced countries does not suit to ours. In advanced countries the general mental level of their students is higher to that of ours. The subject matter we are teaching in higher classes is the subject matter they are teaching in their middle classes. To imitate blindly without proper knowledge is not wise. Hence in our country the circumstances are not in favour of making Mathematics an optional subject after middle classes.

To conclude with we may say that Mathematics should be made compulsory up to high school classes and optional afterwards. At this stage student is capable to decide about their future.

Short in Text Questions

1. What do you understand by Practical value

   ....

   ....

2. How intellectual value developed by Mathemathics

   ....

   ....

1.1.5 Summary

Mathematics is a very useful subject both in daily life and for higher study. It has many educational values but most important from common man's point of view is its practical or utilitarian value. Its other value's like disciplinary value, cultural value, moral value, social value and intellectual value cannot be under estimated. This subject is very useful from not only its use in day to day life rather it is of much importance in the study of higher mathematics, other subjects and in the study of popular literature. In this age of ever advancing society, we have to make pace with other developed societies. The knowledge of today will be out of data tomorrow. So we have to strive hard to acquire power to acquire knowledge. Mathematics helps to develop our intellectual powers and hence power to acquire knowledge. but it will be possible only if progressive, scientific and psychological methods will be used and the teacher, the field worker, will be conscious about their role.

Mathematics should be made compulsory up to high school classes as this is the maximum standard at which the aims of teaching Mathematics are fulfilled.

In the last section of this lesson, we had seen that mathematics plays a dominant and leading role in the development of various physical, biological and social sciences. This relationship shows indirectly the importance of this subject from the study of other subject's points of view.

1.1.6 SELF CHECK EXERCISE

i) The progress and improvement of Mathematics are linked to the prosperity of the state" said by..........................

ii) In the word of.................: "mathemathics is a way to settle in the mind a habit of reasonery.

Ans: i) Napoleon ii) Locke.

1.1.7 SUGGESTED QUESTIONS :-

1. "Mathematics is the mirror of civilization." Discuss.
2. "Mathematics is a science of all sciences and an art of all arts." Comments.
3. What are the values of teaching Mathematics?
4. "Mathematics is a dry subject" How far do you agree with this statement ?

1.1.8 SUGGESTED BOOKS

1. SIDHU (Dr.), K.S. : *The Teaching of Mathematics*.
2. MANGAL, S.K. : *A Text Book of Teaching of Mathematics*.
3. AGGARWAL, S.M. : *Teaching of Modern Mathematics*.
4. KUPPUSWAMI, A.N. : *The Teaching of Mathematics in New Education*.
5. BUTLER AND WREN : *The Teaching of Secondary School Mathematics*.

B.Ed (Semester I) **Paper- IV & V (Opt.-ix)**

Teaching of Mathematics

Lesson No. 1.2 **Author: Mukesh Devi**

Axioms, Postulates, Assumptions and Hypothesis in Mathematics

Structure of the Lesson

1.2.1 Introduction

1.2.2 Objectives

1.2.3 Axioms

1.2.4 Postulates

1.2.5 Assumptions

1.2.6 Hypothesis

- **1.2.6.1 Definition of Hypothesis**
- **1.2.6.2 Hypothesis of Mathematics**
- **1.2.6.3 Statistical hypothesis testing**

1.2.7 Summary

1.2.8 Self Check Exercise

1.2.9 Suggested Questions

1.2.10 Suggested Readings

1.2.1 Introduction

Mathematics is a very important subject. Therefore, before imparting and transmitting its knowledge, it is necessary to understand that, 'What is Mathematics?' Why its knowledge is given? And 'What is its nature? No one definition of mathematic is universally accepted. Generally, there are many definitions of Mathematics for example; some define mathematics as a science of calculation, some as a science of space and numbers and some as a science of measurement, magnitude and direction. the meaning of the word mathematics is – 'The science in which calculations are prime.' In this way on the basis of these assumptions of mathematics, we can say that mathematics is the science of numbers, word, sign, etc. with which we can know about magnitude, direction & space. It is also highlighted in the National Policy on Education (1986), as follows:-

"Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and articulate logically. Apart from being a specific subject it should be treated as a concomitant to any subject involving analysis and meaning."

Mathematics has originated from Numbers and Number System is a special field of it, by which other branches of Mathematics are developed.

Definitions

In Hindi, Mathematics is known as 'GANITA' meaning there by – 'The science of Calculations'. The term Mathematics can be defined in numerous ways to quote oxford dictionary – **"Mathematics is the science of measurement, quantity and magnitude."** Some definitions of mathematics are as follows:-

1. **Marshal H. Stone**- According to Stone, "Mathematics is the study of abstract system built of abstract elements. These elements are not described in concrete fashion."
2. **Bertrand Russell**- According to him, "Mathematics may be defined as the subject in which we never know what we are talking about nor whether what we are saying true."
3. **Benjamin Price** – He emphasized that, "Mathematics is the science that draws necessary conclusions."
4. **Prof**. **Voss** – According to Voss, "Our entire civilization depending on the intellectual penetration and utilization of nature has its real foundation in the mathematical sciences.
5. **According to Galileo** – "Mathematics is the language in which God has written the universe."
6. **According to Locke** – "Mathematics is a way to settle in the mind of children a habit of reasoning."

On the basis of above definitions, we can say or conclude that-

1. Mathematics is the science of space and number.
2. Mathematics is the science of calculations.
3. Mathematics is the science of measurement, quantity and magnitude.
4. Mathematics is a systematized, organized and exact branch of science.
5. It deals with quantitative facts and relationships.

6. It is the abstract form of science.
7. It is a science of logical reasoning.
8. It is an inductive and experimental science.
9. Mathematics is the science which draws necessary conclusions.

1.2.2 Objectives

1. Learners will be able to explain axioms and postulates.
2. Learners will be able to cite examples of axioms, assumptions and postulates.
3. Learner will be able to differentiate between postulates and axioms.
4. Learner will be able to use undefined terms in solving different theorems.

1.2.3 Axioms

Historically the term "axiom" was first used by the Great philosopher Aristotle. According to him, every demonstrative science must start from indemonstrable principle known as first principle. Among these first principles some are peculiar to the particular science but others are common to all sciences. It is these latter first principles common to all sciences which may be termed as axioms. In other words axioms may be known as the common opinions or notions from which all demonstration proceeds and as those things which anyone must told who is to learn anything at all.

In the "Elements", the book written by Euclid these first principles are listed in two categories, the postulates and the common notions. The former are the principles peculiar to the particular science of geometry and the latter, the common notions are evidently the same as Aristotle's axioms. Proclus who has written comments on the "First Book of Euclid" tells us exclusively that the two terms Euclid's "common notions" and Aristotle's "axioms" are synonymous.

Historically therefore, there stands a clear-cut distinction between the terms axioms and postulates. However, a little confusion has been created by some mathematicians in modern times by using the term "postulates" and "axioms" as synonymous But in any way it is quite advantages to maintain distinction between these two terms by keeping reserved the term "axioms" for the axioms of logic or common notions and to use "postulates" for those assumptions or first principles (beyond the principles of logic) by which a particular mathematical discipline (say geometry) may be defined. Thus for the illustrations of the term axioms, we can name the common notions mentioned in the Euclidean geometry.

An **axiom** is a premise or starting point of reasoning. As classically conceived, an axiom is a premise so evident as to be accepted as true without controversy. The word comes from the Greek *axíōma* 'that which is thought worthy or fit' or 'that which commends itself as evident. As used in modern logic, an axiom is simply a premise or starting point for reasoning. What it means for an axiom, or any mathematical statement, to be "true" is a central question in the philosophy of mathematics, with modern mathematicians holding a multitude of different opinions.

In both senses, an axiom is any mathematical statement that serves as a starting point from which other statements are logically derived. Within the system they define, axioms (unless redundant) cannot be derived by principles of deduction, nor are they demonstrable by mathematical proofs, simply because they are starting points; there is nothing else from which they logically follow otherwise they would be classified as theorems. However, an axiom in one system may be a theorem in another, and vice versa.

An "axiom", in classical terminology, referred to a self-evident assumption common to many branches of science. A good example would be the assertion that

When an equal amount is taken from equals, an equal amount results

When mathematicians employ the field axioms, the intentions are even more abstract. The propositions of field theory do not concern any one particular application; the mathematician now works in complete abstraction. There are many examples of fields; field theory gives correct knowledge about them all.

It is not correct to say that the axioms of field theory are "propositions that are regarded as true without proof." Rather, the field axioms are a set of constraints. If any given system of addition and multiplication satisfies these constraints, then one is in a position to instantly know a great deal of extra information about this system.

In the modern understanding, a set of axioms is any collection of formally stated assertions from which other formally stated assertions follow by the application of certain well-defined rules. In this view, logic becomes just another formal system. A set of axioms should be consistent; it should be impossible to derive a contradiction from the axiom. A set of axioms should also be non-redundant; an assertion that can be deduced from other axioms need not be regarded as an axiom.

Common notions

1. Things which are equal to the same thing are also equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

1.2.4 Postulates

Generally in proving a theorem the help of some already proved theorem or theorems is taken. Going backward there must be a theorem for which we have no previous theorem to help. In proving this first theorem we have to accept certain statements without stressing on their proofs. These statements, which are accepted as true in a particular system without stressing on their proofs, are labeled as postulates. Meaning thereby that every system has its own selection of postulates i.e. the unproved first principles. For example in case of a system in arithmetic adopting “0” (zero), “number” and “successor” as the undefined or primitive terms, the related postulates may be named as below:

(1) 0 is a number
(2) Every member has a number as its unique successor
(3) Two members having the same successor are identical
(4) If 0 belongs to a class F, and if whenever a number x belongs to F, the successor of x belongs also to F, then all numbers belong to F.
(5) 0 is not a successor of any number

At the foundation of the various sciences lay certain additional hypotheses which were accepted without proof. Such a hypothesis was termed a *postulate*. While the axioms were common to many sciences, the postulates of each particular science were different. Their validity had to be established by means of real-world experience. Indeed, Aristotle warns that the content of a science cannot be successfully communicated, if the learner is in doubt about the truth of the postulates.

The classical approach is well-illustrated by Euclid's Elements, where a list of postulates is given (common-sensical geometric facts drawn from our experience), followed by a list of "common notions" (very basic, self-evident assertions).

(1) A line is a set of points contains at least two points.
(2) If two planes contain a point in common, they have another point in common.
(3) A plane contains at least three distinct non-collinear points.
(4) It is possible to draw a straight line from any point to any other point.
(5) It is possible to extend a line segment continuously in both directions.
(6) It is possible to describe a circle with any center and any radius.
(7) It is true that all right angles are equal to one another.
(8) ("Parallel postulate") It is true that, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, intersect on that side on which the angles are less than the two right angles.

In any system of geometry Euclidean or Non-Euclidean postulates make it possible to prove several theorems with great rigor and with no reference to drawings. Every system in all the branches of mathematics is known to have its own sets of assumptions and postulates.

1.2.5 Assumptions (undefined or primitive terms)

It is very essential to know for a mathematics student that it is not possible to define everything. One has to start from some undefined terms. For example in arithmetic the undefined or primitive terms are "0" , "number" (in the sense of non-negative number whole number) and "successor" (in the sense that x + 1 is the successor of x). Similarly in geometry, the terms, points, line and surface and in algebra, set, number and variables have been accepted as undefined or primitive terms. It should also be made clear that mathematician is free to fix undefined terms according to the system. For example if one accepts between's as undefined term then he may be able to give a definition of the line also.

When doing any sort of math word problem, you usually have to make a few assumptions in your calculations. Otherwise a question may be too complicated to solve.

For example, if I'm asked to find the area of rectangular garden plot that's 6 feet by 4 feet, I have to ASSUME that 1) the plot is perfectly rectangular (even though in reality one of the corners may be off by a couple of degrees), 2) the measurements are exact (even though in reality it's easy to misread a tape measure by 1/16th of an inch, and a metal tape measure might expand a bit in the heat, etc.). This way I can do the calculation and get 24 square feet. So in this case, you're making a lot of assumptions, maybe without even realizing it. You're assuming that:

1) The camping site can be thought of as a single point. Even though two people who are sitting even an inch apart will view the angles a little differently.
2) The possum doesn't move while the owl is diving after it.
3) The owl was flying at a constant speed

1.2.6 Hypothesis

Hypothesis is the part of a conditional statement just after the word if. Etymologically, hypothesis is made up of two words, "hypo" (less than) and "thesis", which means less than or less certain than a thesis. It is the presumptive statement of a proposition or a reasonable guess, based upon the available evidence, which the person seeks to prove through his study. The hypothesis is precisely defined as a tentative or working proposition suggested as a solution to a problem.

1.2.6.1 Definitions of hypothesis

(1)Acc to **James E. Garton:** "Hypothesis is a tentative supposition or provisional guess which seems and explains the situation under observation."

(2)Acc to **John W. Best**: "It is shrewd guess or inferences that is formulated and provisionally adopted to explain observed facts or conditions and to guide in further investigation."
(3)Acc to **Good and Hatt:** "A hypothesis states what we are looking for. A hypothesis looks forward. It is a proposition which can be put to a test to determine its validity. It may prove to be correct or incorrect."

Examples of Hypothesis
In the conditional, "If all fours sides of a quadrilateral measure the same, then the quadrilateral is a square" The hypothesis is "all fours sides of a quadrilateral measure the same".

Solved Example on Hypothesis

Questions: In the example above, is the hypothesis "All fours sides of a quadrilateral measure the same" always, never, or sometimes true.

Choices:
A. Always
B. Never
C. Sometimes

Correct Answer: C

Solution:
Step 1: The hypothesis is sometimes true. Because, its true only for a square and a rhombus, not for the other quadrilaterals rectangle, parallelogram, or trapezoid.

1.2.6.2 Hypotheses and mathematics

A hypothesis is the "if" part (antecedent) of a conditional statement.

In mathematics, a *hypothesis* is an unproven statement which is supported by all the available data and by many weaker results. An unproven mathematical statement is usually called a "conjecture", and while experimentation can sometimes produce millions of examples to support a conjecture, usually nothing short of a proof can convince experts in the field. But when a conjecture is supported not only but all the available data but also by numerous weaker results, it is upgraded in label to a hypothesis.

So where does mathematics enter into this picture? In many ways, both obvious and subtle:

- A good hypothesis needs to be clear, precisely stated and testable in some way. Creation of these clear hypotheses requires clear general mathematical thinking.
- The data from experiments must be carefully analyzed in relation to the original hypothesis. This requires the data to be structured, operated

upon, prepared and displayed in appropriate ways. The levels of this process can range from simple to exceedingly complex.

Very often, the situation under analysis will appear to be complicated and unclear. Part of the mathematics of the task will be to impose a clear structure on the problem. The clarity of thought required will actively be developed through more abstract mathematical study. Those without sufficient general mathematical skill will be unable to perform an appropriate logical analysis.

1.2.6.3 Statistical hypothesis testing

Essentially, a hypothesis is accepted when the chance of it being correct is 'high' and the chance of it being incorrect is 'low'. Whilst in our interactions with the physical world we use our intuition to create hypotheses, for certain numerical situations we can quantify the words 'high' and 'low' and then determine which category the data fall into. Rather than being vague, the mathematical theory of *probability and statistics* allows us to be very precise in these assessments. Once we have precisely described our situation, hypotheses are no longer simply accepted as true or rejected as false but are accepted or rejected with some **degree of certainty**. A precise understanding of such ideas requires a good grasp of risk, probability, randomness, statistics and distributions. Statistics shows that there are certain patterns in randomness and that underlying the concept of randomness are a number of key ideas or objects, such as the *normal distribution*. Once we have grasped such issues, we can begin to make exact mathematical statements similar to these

- If a set of data is normally distributed with mean 0 and variance 0.25 then there is a 99.94% certainty that a measurement will not exceed 1.0.
- If the sample mean of a set of data is 12, how confident can we be that the theoretical mean lies between 11 and 13?

It is at this point that making and testing hypotheses becomes a true branch of mathematics. This mathematics is difficult, but fascinating and highly relevant in the information-rich world of today.

Perhaps it would be more appropriate to give the definition of "hypothesis" that is often used in mathematics (i.e., a synonym is "assumption"). Or maybe this other definition of "hypothesis" should be a separate entry.

Short in Text Questions

1. Differentiate between Axioms and postulates

   ………………………………………………………………………………………

2. Define Hypothesis of Mathematics

   ………………………………………………………………………………………

1.2.7 Summary

Knowledge of undefined terms, postulates and axioms is requirements for going ahead with deductive reasoning of mathematics.

Knowledge of assumptions is necessary for a mathematics student because it is not possible to define everything. One has to start from some undefined terms. We can only assume these but cannot be proved.

The statements, which are accepted as true in a particular system without stressing on their proofs labeled as postulates.

Historically therefore, there stands a clear cut distinction between the terms axioms and postulates. However, a little confusion has been created by some mathematicians in modern times by using the terms “postulates” and “axioms” as synonymous. But in any way it is quite advantages to maintain distinction between these two terms by keeping reserved the term “axioms” for the axioms of logic and to use “postulates” for those assumptions or first principles by which a particular mathematical discipline may be defined. Thus for the illustrations of the term axioms, we can name the common notions mentioned in the Euclidean geometry.

1.2.8 Self Check Exercise

Match the following

1)	Mathematics is the science that draws necessary conclusion	a) Galilea
2)	Mathematics is the language in which God has written its universe.	b) Locke
3)	Mathematics is a way to settle in the mind of children a habit of reasonary.	c) Benjamin Price

Answer: 1) –C 2) –a 3) –b

1.2.9 Suggested Questions

1) Describe Assumptions terms with suitable examples and why are these important in mathematics?
2) Explain concept of mathematics. Give detail of axioms with examples.
3) Describe the significance of postulates and assumptions in teaching of mathematics with the help of examples.
4) Write short note on the following :
    1) Axioms
    2) Postulates

3) Assumptions
4) Hypothesis in mathematics

5) What is difference between an axiom and a postulate?
6) What are functions of axioms, assumptions and postulates in teaching of mathematics?
7) Define hypothesis. Give some examples of hypothesis in mathematics.
8) Which type of reasoning is used in hypothesis testing?
9) Discuss the need and importance of hypothesis in mathematics.

1.2.10 Suggested Readings

1) Dave,R.H. and Saxena,R.C.: “Curriculam and teaching of maths in sec. school”, New Delhi:Research Monograph,NCERT,1970.
2) Mangal,S.K.: “Teachinh of Mathematics”, New Delhi: Arya Book Depot,2008..
3) Rawat,M.S.: “Teaching of Mathematics”, Agra:Vinod Pustak Mandir.
4) Sidhu, K.S. : “Teaching of Mathematics”, New Delhi sterling Publication, 1990
5) Kumar, Sudhir and Ratnalikar, D.N. : “Teaching of Mathematics”, New Delhi: Anmol Publications, 2003
6) Dalal,D.C.: “Teaching of Mathematics”, Ludhiana: Vijay Publications
7) Mangal,S.K.: “Teaching of Mathematics” Ludhiana: Parkash Book Depot
8) Kulsheshetha, A.K: “Teaching of Mathematics” Meerut, R. Lal Book Depot.
9) Aggarwal, Sunita: “Action Research.”Ludhana: Vijay Publications.
10) Kaul,Lokesh: “Methodology of Educational Research”, Third Revised and Enlarged Edition, New Delhi:Vikas Publishing House Pvt LTD,2006.
11) Aggarwal,Y.P.: “The Science of Educational Research.” Kurukshetra :Nirmal Book Agency, first published 1998, Edition 2008.

B.Ed (Semester I) **Paper- IV & V (Opt.-ix)**
Teaching of Mathematics

Lesson No. 1.3 **Author: Mukesh Devi**

Historical development of notations

1.3.1 Objectives
1.3.2 Introduction
1.3.3 Value of History of Mathematics
1.3.4 How and Why Mathematics Originated in India
1.3.5 Historical Reviews of Developments
- **1.3.5.1 Historical Development of Notion System**
- **1.3.5.2 Historical Development of Metric system of Weights and Measures**
- **1.3.5.3 History of the Development of Logarithms**
- **1.3.5.4 Historical Development of computer Mathematics**

1.3.6 Summary
1.3.7 Self check Exercise
1.3.8 Suggested Questions
1.3.9 Suggested Readings

1.3.1 OBJECTIVES

1. Learners will be able to define notation and metric system.
2. Learners will be able to know about the evolution of number system.
3. Learners will be able to explain the historical development of notation, metric system, logarithms and computer mathematics.
4. Learners will be able to compare the number system of different countries.
5. Learners will be able to cite examples of notation system, metric system, logarithms.

1.3.2 Introduction

Mathematics has been the backbone of our civilization. It is no exaggeration to say that history of Mathematics is the history of civilization. Mathematics has led to the development of various subjects, and technology. Mathematics is an exact science which is still playing an important role in various walks of life.

According to J.W.L. Glaisher, "I am sure that no subject looses more than Mathematics by any attempt to dissociate it from its history."

Mathematics has been a progressive science.

1.3.3 Utility of History of Mathemathics

For a teacher the utility of history of Mathematics can be summarized as under:

- (i) It provides a glimpse of the role played by Mathematics in various walks of life.
- (ii) It shows the correlation of Mathematics with other branches of knowledge.
- (iii) It clearly brings out the fact that significant developments in the subject have always been prompted by human needs.
- (iv) It makes clear the fact that Mathematics is a manmade science.
- (v) It makes the teaching of Mathematics interesting.
- (vi) It is easier to understand most of the mathematical topics, concepts and terms by reference to their historical background.
- (vii) It helps in gradation of subject matter; the topics discovered earlier were simpler.
- (viii) It makes the correlation of Mathematics with other subjects easier.
- (ix) It helps in enhancing the reputation of the teacher.
- (x) It provides a warning against making hasty conclusions.
- (xi) It gives better understanding of the subject.

The history of Mathematics depicts the different stages in the development of Mathematics throughout the ages. It shows that Mathematics is a dynamic subject, not static .

1.3.4 How mathematics organized in India:

Everyone recognizes that the earliest contribution of India to this field was the ten-base system but actual dates are not known. In order to keep this section factual, I will restrict my descriptions to mostly what is historically known. However, as new information becomes available, this section will necessarily need revision. We now know that at least a large city-dwelling population existed in a place called Dwarka (which is now submerged off the coast of Gujarat approximately 9,500 years ago). If recordings from this city show mathematical notations, clearly the story of mathematics will become that much older.

What is historically known goes back to the days of the Harappa civilization, 2600-3000 BCE. Since this Indian civilization delved into commerce and cultural activities, it was only natural that they devise systems of weights and measurements. For example a bronze rod marked in units of 0.367 inches was discovered and point to the degree of accuracy they demanded. Evidently, such accuracy was required for town planning and construction projects. Weights corresponding to 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200 and 500 have been discovered and they obviously played important parts in trade and commerce development.

The next phases of mathematics developments are found in Vedic texts, along with ritual practices, and in the Puranas. Calculations for the precise building of ritual altars would have been important, for obvious reasons. Arithmetic principles such as addition, subtraction, multiplication, fractions, cubes, squares and roots were developed during these periods (Narad Vishnu Purana (by Veda Vies ~1000 BCE)). Geometric principles ("Rekha-ganit) are found in the Sulva Sutras, authored by Baudhayana (800 BCE) and Apasthamba (600 BCE). Calculations of one geometric shape (for example a rectangle) and converting them to an equivalent (or fraction or multiple thereof) square or circle would require a good working knowledge of geometry. Pythagoras, who studied mathematics in India (6th century BCE), became familiar with the principles of geometry from the Sulva Sutras. Even the so-called Pythagoras theorem was described by Baudhayana in his Sutra. ("The chord that is stretched across the diagonal of a square produces an area of double the size"). His Sutra also contained geometric solutions of a linear equation in a single unknown and even examples of quadratic equations. Apasthamba Sutra described calculation of the value for the square root of 2, accurate to the fifth decimal place. A solution to the general linear equation is also described in this Sutra and the squaring of a circle and dividing a segment into seven equal parts.

1.3.5 Historical reviews of development

In the history of Mathematics there have been some very remarkable developments in the form of discovery and evolution of certain ideas and processes. These ideas and processes claim special status and significance in the overall progress of mathematical knowledge. They are considered to the landmarks in the history of the subject. At this moment we shall discuss only a few of them

1.3.5.1 Historical development of notation system

Number sense is as primitive as civilization itself. Even birds and insects are supposed to have some number sense. As the history reveals, primitive man did not possess a number sense higher than that possessed by animals, birds and insects. He could know what was meant by one animal or two but could not tell one and one makes two. He could express more or less objects with the help of his hands and arms movements. He was not able to name the objects with numbers. If a shepherd had to say that he had five sheep he could tell it with the same number of pebbles on the bases of principal "One-to-one correspondence."

It took a long time for the man to name the numbers. He started to name them with the help of language spoken by him. Up to this time he was no able to write these numbers. He had not learnt the use of signs or symbols for the numbers, for example to use numeral "2" instead of the word "two".

Where and when did notation (use of number symbols) begin? This question takes us back to the very beginning of the history. At least as back as 3000 B.C., Egyptians, Sumerians, Babylonians, Greeks, Romans, Chinese and Indians tried to put their steps in developing some sort of notation system for expressing numbers of different values from units up to hundreds of thousands.

The numeral '1' perhaps meant one lifted finger.

'Two' was represented by two fingers or lines. If we write two lines without lifting the pen, it becomes Z which ultimately changed into 2.Or It becomes µ the numeral used by the Arabs or Persians. Similarly if we draw three lines without lifting the pen, it becomes 3 or ຕ̄ or ३ of all numerals for three. The Babylonians used wedge-shaped symbols. One was represented by V, ten was represented by < and hundred by V <.The Roman system is based on the idea of counting fingers or lines. Thus I, II and III represented one, two and three respectively V probably represented the whole hand. To avoid clumsy I I I I they wrote I before V i.e. IV, the symbol gave rise to the idea of positional value. Then the symbols VI, VII, VIII etc. The symbol X was perhaps the combination of two fives.

The origin of the rest of the numerals is generally unknown. Since in most countries, the religious leaders of the community like priests and saints were practically the only educated persons and since there were no ways of communications, the different country men or even tribal sin the same country or region developed sets or different culture language. As a result each one of the social group or race developed its own letters and numerals through its Priests or Pandits (scholars). Therefore, the numerals developed where too diversified. Later on when the traders, merchants and rulers used to cross the boundaries, the numerals tended to become more alike.

The difference in the presentation of numerals at times by the different people may be visualized through the following presentation.

(1) Greek Enumeration and Basic Number Formation

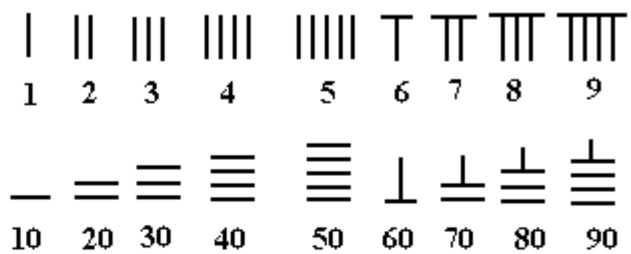
First, we note that the number symbols were the same as the letters of the Greek alphabet.

symbol	value	symbol	value	symbol	value
α	1	ι	10	ρ	100
β	2	κ	20	σ	200
γ	3	λ	30	τ	300
δ	4	μ	40	υ	400
ϵ	5	ν	50	ϕ	500
digamma.gif	6	ξ	60	χ	600
ζ	7	o	70	ψ	700
η	8	π	80	ω	800
θ	9	koppa.gif	90	sampi.gif	900

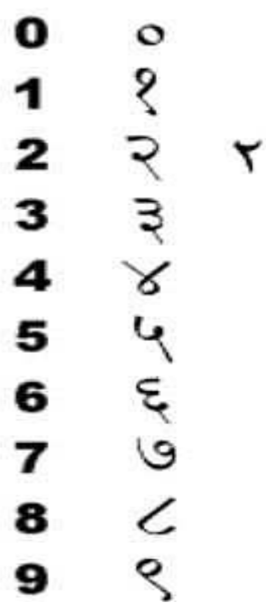
where three additional characters, the ϛ (digamma), the ϙ (koppa), and the ϡ (sampi) are used. Hence,

$$\sigma\pi\zeta = 287$$

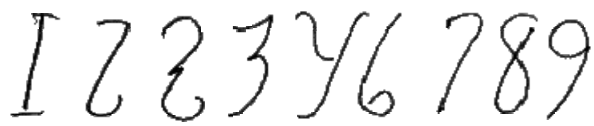
(2) oldest numerals known to be used bye the chinese



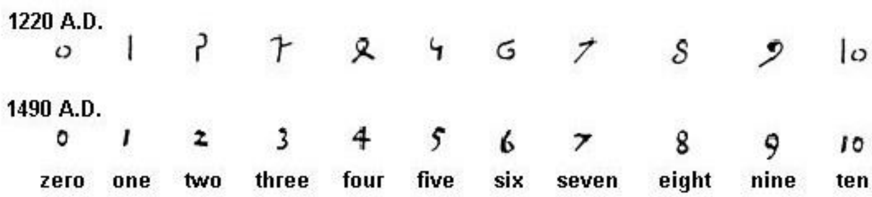
(3) Hindu numerals which were taken to Arab about a thousand years ago



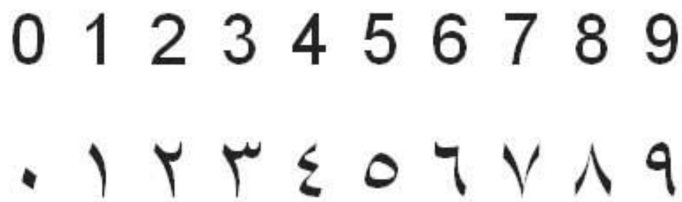
(4) Numerals used in Europe in the year 976 A.D.



(5) Numerals used in Europe in the year

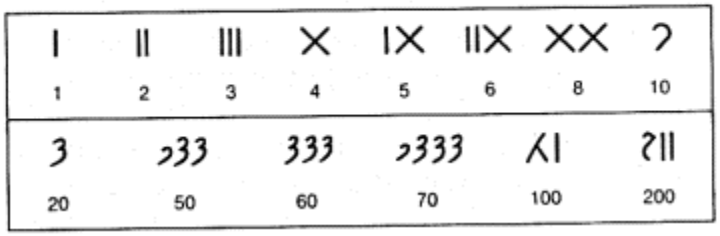


(6) Arabic numerals used then and now

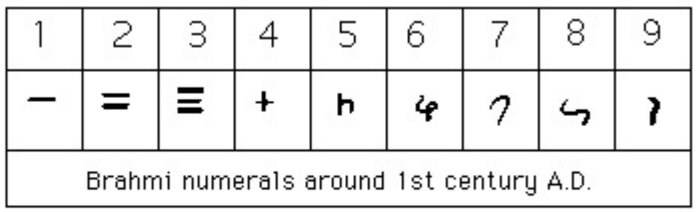


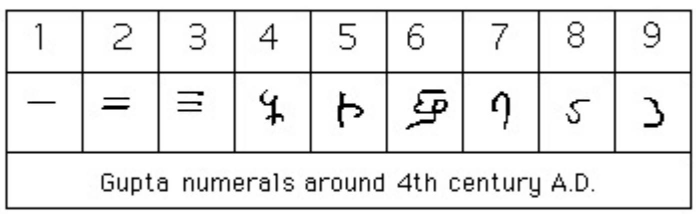
"Arabic'' numerals are believed to be of Indian origin, although not all specialists agree with this hypothesis. Numbers are first encountered in Indian writings in the third century B.C. At this time two forms of writing were used, Kharoshti and Brahmi, and each one had its own numerals. The Kharoshti system is interesting because the number four was selected as the intermediate stage between 1 and 10. It is likely that the oblique cross (x) used as a 4 tempted the creators of the Kharoshti numbers by its simplicity of writing while still preserving the modeling quality in full (four rays). The Brahmi numerals are more economical. It is believed that the first nine Brahmi characters finally gave rise to our modern numerals.

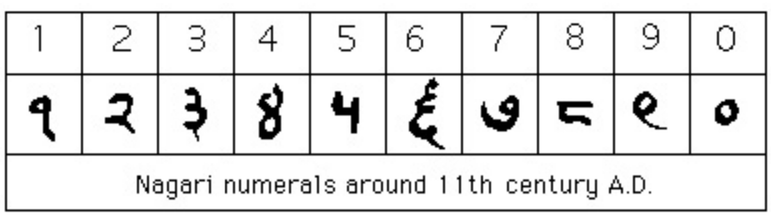
Kharoshti numerals:



Indian number system





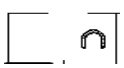


(7) 1 2 3 4 5 6 7 8 9 10 (the present form of the numeral)

(8) the roman numerals

	1	2	3	4	5	6	7	8	9
Ones	I	II	III	IV	V	VI	VII	VIII	IX
Tens	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
Hundreds	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
Thousands	M	MM		-	-	-	-	-	-

Later on combining numerals and writing large numbers presented a new problem before mathematics. The scheme of grouping the objects was planned and symbols were invented for the group of five, ten, hundred, thousand etc. Some of such symbols may be seen in the Chinese and Roman numerals given before. The different people invented different symbols at times at may be seen from the following presentation of the numerals for "ten":-
Egyptian, Babylonian, Ancient Rome, Chinese, Indian, Malaya, English.

		x		10	=	10

The problem of writing large numerals still remained unsolved. How cumbersome was the task, may be visualized through the examples given ahead:

1. Writing of the number 3287 by Greeks.

$$^{\prime}\gamma\sigma\pi\varsigma = 3287$$

2. Writing the number 2, 752,899 by the Romans.

CD CDDCCLCDCDDCCCLXXXXCVIIII

In this way large numbers posed a great difficulty in writing. This difficulty was not overcome until Indian mathematics appeared on the scene with their two great mathematical inventions namely zero and place value system. These inventions played a miracle by making possible the writing of any number (no matter how large it could be) with the help of any ten symbols , 1,2,3,4,5,6,7,8,9 and 0. Commenting on this great achievement, the great French mathematician Laplace (1749-1827) writes :-

"India has offered us a system for expressing all numbers with the help of only ten symbols………… This Indian invention picks up its new height when we recall that the most talented great mathematicians like Archimedes and Apollonius failed to discover such simple but essential system."

1.3.5.2 Historical development of metric system of weights and measures

The metric system was first described in 1668, and officially adopted by France in 1799. Over the course of the 1800s and 1900s, it became the dominant system worldwide including the only measurement system enacted by law by the United States. Numerous countries continue to use their customary units. The American system is unusual, however, in not having adjusted itself to close metric values in the manner of the Scandinavian mile (now 10 km exactly), the Chinese jinn (now ½ kg exactly) or the Dutch ones (now 100 gr).

Primitive man used different stones, seeds etc. for the purpose of weighting. In India, Ratti (a seed) was taken as the basic unit of weight, 'Penny' an English unit of weight was considered as equal to 32 wheat seeds, various limbs of body were used by man to measure lengths e.g. cubit, span, pace, foot etc. In England a yard was fixed as the distance from the nose to the thumb of King Henry I.

The history of the development of the system of weights and measures is as old as the civilization itself. In the beginning there were no weights and measures. There was no need of any currency. People used to exchange various commodities as and when they required such exchange. The measurement of length was in a crude form. On filling the necessity of length measurement it appears that man tried to take help of his own body parts. As a result the measurement like 'Cubit', 'Span' 'Digit' ,'Palm' and 'Step' were adopted.

Because of lack of communication systems the units of weights and measures varied from place to place. The unit 'Pound' was used by

Romans. At some places the 'pound' was considered to be of 13 ounces at another of 18 ounces. So there was a lot of confusion in the system of weights and measures. In 1960 the Eleventh General International Conference on Weights and Measures adopted a system of units that includes the original Metric units and various other units used in Science and Engineering. (For details refer *Encyclopedia Britanica* please).

In this way, it may be seen that the primitive means of weighing and measuring were very crude. These weights and measures did not have adequate reliability or validity. They were not standardized. But as the time passed, there came improvement in the system of weights and measures. For exchange of commodities, coins were introduced consequently different types of coins and systems of weights and measures were developed in different countries.

In this way there were many different systems of measuring and weighing before the introduction of the metric system not only in different countries but often in each country itself, for in France alone there were at one time eighteen different ways of measuring cloth. With the development of communication between countries it was realized that these differences should be removed as they hamper the development of trade.

It was thought that there should be a universal system of weights and measures in different countries and attempts were made in his direction. In 1670, Gabriel Mouton, a clergyman of France for the 1st time proposed a decimal system having as its basis the length of an arc, one minute of the earth's circumference. He suggested this unit should be called a milliareor mile and that larger and smaller measurements could be determined by multiplying this by tens or by subdivision by tens. The other suggestion was that of taking the length of the pendulum which was beating seconds or half seconds. These ideas were taken so seriously during the next 100 years that in 1789, the French Academy of science investigated the whole matter with the later encouragement of the French parliament.

The idea of using the length of a pendulum was discarded in favour of taking a line on the earth's surface. A survey was made of the length of a line from the North Pole to the equator running through Dunkirk in France and Barcelona in Spain.

This distance was divided into ten million parts and one of these was called a matter. This was equivalent to 39.37 inches and it became known as the standard meter.

For small measurement the meter was divided by multiples of ten, and for large measurement it was multiplied in the same way. While dividing it was decided to give a Latin prefix to the new part, deci, centi, milli, and when multiplying to use Greek prefixes, deca, Hecto- and kilo .

It was very simple matter to use the meter and its parts for areas and volumes. But for the liquid measures a cubic vessel of exactly ten centimeter on each side was used, so that its volume was a thousand cubic centimeters the amount of water this vessel would hold was then called a liter and this again could be subdivided and multiplied into deciliters or deciliters etc.

For standard units of weight one cubic centimeter of pure water at 4 degree centigrade was used. This was called the gram, a platinum cylinder known as the kilogram of the archives was declared as the standard for one thousand grams. For weight also the same prefixes were used for the multiplication and division as with length and capacity.

In 1840 this system, the only scientific method of weighing and measuring in the world, was made compulsory in France. In 1874 however, a new arrangement called the Metric Treaty set up a new standard for the meter and the kilogram. The new meter consisted of a bar of 90 % platinum and 10% iridium was called as the 'International prototype meter'.

When the 1st kg was made it was impossible to measure the volume of water of a cubic decimeter to a millionth part and because of this there is not now the comparison between the meter , the kilogram and the litre that was intended in the 1st place . If a certain volume of water is measured by linear dimensions and it is also weighed the two results do not coincide exactly but differ by 28 parts in 1,000,000. The litre is, therefore now defined as the volume of a kilogram of pure water at its maximum density and under standard pressure, and it is equivalent to 1,00028cubic deci-meters, though for most practical purposes this slight difference may be ignored.

The next stage in the history of the metric system was reached in 1927 when an international conference on weights and measures adopted an additional definition of the meter in terms of the wave-length of the light. This system proved so useful and convenient that most of the countries began to adopt decimal system of coinage. Sense of the enthusiastic scientists of France even suggested the following change in measures of time:-

100 seconds = 1 minute
100 minutes = 1 hour
10 hours = 1 day.

But this idea could not proceed ahead due to its non-practicability. Similarly one more idea of dividing a right angle into 100 parts and circle into 400 degrees could not also move ahead.

Hindu units of time on a logarithmic scale

The division of the circle into 360 degrees and the day into hours, minutes, and seconds can be traced to the Babylonians who had sexagesimal system of numbers. The 360 degrees may have been related to a year of 360 days. Many other systems of measurement divided the day differently -- counting hours, decimal time, etc. Other calendars divided the year differently.

The metric system is now used by almost every civilized country. In the beginning this system also had to face though opposition and criticism from its opponents. England did not adopt it due to its enmity with France and even fixed a fine of five pounds on the citizen who were found to use metric system of weight and measures. But at last it could not deny this system its dew place and in 1877 Queen Victoria had to declare it as legal measure. Since then its use had been increasing day by day. Nearly all the common-wealth countries have adopted it. In our country also, this system has been formally adopted since April 1, 1957.

1.3.5.3 History of the development of logarithms

The wonderful powers of modern calculation owe their debt to three inventions—Hindu notation Decimal Fraction Logarithms. There is nothing in Mathematics of previous centuries which might be said to have led to this invention. Borrowing the words of Lord Moulton: "The invention of logarithms came on the world as a bolt from the blue. It is a solitary exceptional mathematical invention that has resulted from the work of any 1 individual."

This individual who is credited for the invention of logarithms was John Napier (1550-1617) the famous mathematician and citizen of Scotland. In 1614 he published his "Mirifice Logarithmorum Canonis Description" in which he explained his famous invention. In its preface he wrote: "seeing there is nothing that is so troublesome to mathematical practice, nor both more molest and hinder calculators, then the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors. I began therefore to consider it my mind by what certain and ready art I might remove those hindrances."

Napier's table of logarithms

Arithmetic	**1**	**2**	**3**	**4**	**5**	**6**	**...**	**Logarithms**
Geometric	**2**	**4**	**8**	**16**	**32**	**64**	**...**	**Antilogarithms**

To the preparation of this little book of 147 pages Napier had devoted twenty years of service steady labour and his labour was proved too fruitful to save untold hours of time consuming mechanical computation. By using logarithms and difficult process of multiplication and division were changed to that of simple addition and subtraction and the extraction of any root could be performed by a simple process, division. Now the question arises from where did John Napier get the idea of conversion. Most probably he might have caught the idea from the Trigonometrical formulae such as sin A sin B =1/2 [cos(A-B)—cos(A+B)] in which the product of two trigonometrical functions can be expressed as the sum of difference of two other functions.

Henry Briggs (1561-1631), professor of mathematics in London was much impressed by Napier's invention. He made the difficult journey northwards for the purpose of meeting Napier and discussing with him about the new invention. Briggs put his point of view that is would be much more convenient that zero should be retained as the logarithm of the whole sine as suggested by Napier, but that the logarithm of the 10^{th} part of the whole sin or 5deg. 44 min. 21 sec. should be 10,000,000,000.

As they talked over the change in the logarithms, Napier said that he was himself thinking for the change but his suggestion was that the change should be effected in this manner, that zero should be the logarithm of unity and 10,000,000,000 that of the whole sine. This would make the logarithm of all numbers greater than unity +ve. Briggs readily agreed to the suggestion. It is clear from the table of logarithms published by Briggs that the system finally decided upon was one in which the logarithm of 10 was unity and the logarithm of unity was zero.

In this way a system of logarithms to base 10 was developed, the system today known as common logarithms, to distinguish them from Napierian logarithms. In fact it was the beginning of the present 1to 1000 to 14 decimal places and got his results published in 1617 in a pamphlet of 1617 pages 'Logarithmorus Chilias Prima.' After then Briggs took gigantic task computing the logarithms of the numbers from 1 to 100,000 of fourteen places of decimal. His arithmetic Logarithmica Sive Logarithmorun Chiliades Triginta containing the logarithms of the 1^{st} 20,000 and the last 100,000 numbers appeared in 1624. Later on the logarithms of the intervening numbers were published with the help of a Dutch bookseller, Advian Valacq in 1628, Biggs and Valacq jointly published for complete log tables of 14 places. For many years these tables were used as the only table of largest places. This monopoly was broken between 1924 and 1949 by England after publishing log tables of 20 places.

The only mathematician of time who had grapsed the principal upon which logarithms were based and who tried to construct log tables was the Swiss Joost Burgi (1553-1632). His 'Arithmetics Sche Und Gometic Sche progress –Tabulex' was published in 1620, 6 years after the appearance of Napier's description. His tables may be considered as modified tables of antilogarithms, the base 1.0001. But the credit for the rapid adoption of logarithms throughout Europe goes largely to the enthusiasm of Briggs.

Briggs devoted the closing years of his life to the calculation of the trigonometrical functions. The work was completed by his colleague at Greshman College, Henry Gellibrand and eventually appeared in 1633 under the tital 'Triangalorum Britannica Sive Doctuna Triangalorum'.

Meanwhile Valacq published his Trigonometrica Artificiates which gives the logarithms of sines, tangents and Secants for every 10 second to seven places . He for the 1st time, made reference to the principal upon which common logarithms are based like those given ahead.

Number	logarithms
1	0.000000
10	1.000000
100	2.000000
1000	3.000000
10000	4.000000

The enthusiasm for the new methods was further carried on by Gregory and Mercator who devised new methods of constructing logarithms in the year 1667 and 1668.

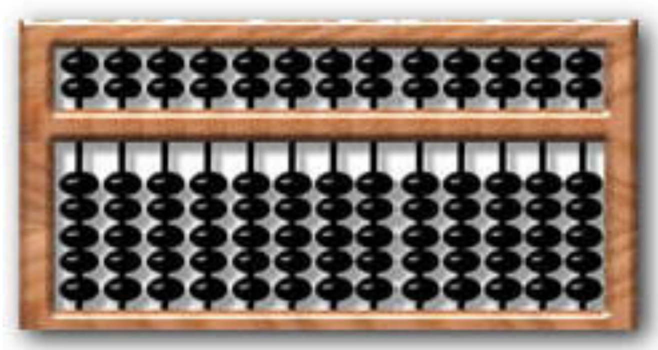
It is also interesting to know that the word logarithm was, first of all, used by John Napier. In the beginning he was using the term artificial number of this purpose. The term 'Mantissa' was introduced by Henry Briggs. It is a Latin word meaning connecting or adding. The word 'Characteristics' was also given by Briggs. Historically it is also important to mention that in the log tables published in the old books, Mantissa as well as Characteristics both were mentioned. The new system in which Mantissa is mentioned is the contribution of 19^{th} century.

1.3.5.4 Historical development of computer mathematics

Since its 1^{st} acquaintance with the world of numbers, man has been trying hard to find out the possible means of computation with utmost speed and greatest accuracy. In real sense human mind is a great computer but it has also its limitations. Therefore man has always tried to invent some or the other devices for enabling his mind to compute efficiently. Electronic is the latest invention in this direction.

Abacus : The First Computer of The world

The development of computer mathematics has its own history. The first computer used for computation was the use of fingers of hand and foot. The second step was the invention of 'abacus'. This apparatus consist of a frame containing a number of wood or metal beads in several wire. Any number may be presented by sliding these beads.



In China and Japan is still in use of carrying out the operations of addition, subtraction, multiplication and division.

John Napier's Rods says that the next important development in mechanical means or calculation was that introduced by the founder of modern logarithms, John Napier, in 1617. He arranged a series of rods.

Method of Use

Suppose it is required to multiply 364 by 4. The columns headed 3, 6 and 4 are selected and placed side by side in that order.

As the multiplier is 4, the fourth line is read off by adding the number between the oblique parallel lines which according to figure become 1456, the correct answer.

These rods were extremely popular, not only in Europe but even in China and Japan and for many years they occupied an important place in mathematics. As it may be guessed, they could also be used for division though the procedure was a little less straight-forward.

Short in Text Questions

1. Define Notation System

   ……………………………………………………………………………………

   ……………………………………………………………………………………

2. Explain Historical development of Matric System.

   ……………………………………………………………………………………

   ……………………………………………………………………………………

1.3.6 Summary

The most fundamental contribution of ancient India in mathematics is the invention of decimal system of enumeration including the intervention of zero. In the beginning of civilization, a man was an uncivilized person. He had no knowledge of numbers. He could know what was meant by one or two animal but could not tell one and one make two or express in numbers. He could express more or less objects with the help of his hands and arms movements. It took a long time for the man to name the numbers. The decimal system uses nine digits (1 to 9) and symbol zero to denote all natural numbers by assigning a place value to the digits. The arabs carries this system to Africa and Europe. Hindu numerals which were taken to arab numerals. The inventions of zero and decimal place value played a miracle by making possible the writing of any number with the help of any ten symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.

The metric system is now used by almost every civilized country. In the beginning this system also had to face tough opposition and criticism from its opponents. England did not adopt it due to its enmity with France and even fixed a fine of five pounds on the citizen who were found to use metric system of weights and measures. But at last it could not deny this system its due place and in 1877 Queen Victoria had to declare as legal measure. Since then its use has been increasing day by day. Nearly all the common-wealth countries have adopted it. In our country also, this system has been formally adopted since April 1, 1957.

The word logarithm was used by John Napier firstly. In the beginning he was using the term artificial number for this purpose. The term 'Mantissa' was introduced by Henry Briggs. It is a L atin word meaning connecting or adding. Historically it is also important to mention that in the log tables published in the old books, Mantissa as well as Characteristics both mentioned is the contribution of 19^{th} century.

Computer technology begun from a simple calculating device abacus and developed into on electronic device known as scientist's play thing can now be adopted to almost any use including that of operating quite advanced and sophisticated machines. The development electric brain is able is to solve in a few minutes, equations which would take the most brilliant scientist several months if not years.

1.3.7 Self check exercise

1. …………………. Numerals are believed to be of Indian origin.
2. The matric system was first described in………………….
3. John Napies, the famous mathematician, citizen of …………….

Ans: 1. Arabi 2. 1668 3. Scotland

1.3.8 Suggested Questions

1) Write an essay on history of mathematics?

2) Describe the historical background of the development of notion system.
3) Describe the development of notion system. What are the characteristics of hindu – arabic system of notation and how is superior to other system?
4) Write short note on the historical development on following topic :
   a) Notation system
   b) Metric system
   c) Logarithms
   d) Computer mathematics
5) Trace the historical background of computer system.
6) Describe historical development of metric system.
7) Discuss the evolution of logarithms.
8) Describe the development of metric system of weights and measurements. How is the system of an improved on other such system? Illustrate by examples the convenience it has created in mathematics in mathematical calculations
9) Discuss the importance of notation and metric system.
10) Discuss the significance role of logarithms and computer mathematics in present time.

1.3.8 References

1) Cajori,F: "A History of Mathematics", New York : MC Millian Pub. Comp,1961.
2) Dave,R.H. and Saxena,R.C.: "Curriculam and teaching of maths in sec. school", New Delhi:Research Monograph,NCERT,1970.
3) Mangal,S.K.: "Teachinh of Mathematics", New Delhi: Arya Book Depot,2008..
4) Rawat,M.S.: "Teaching of Mathematics", Agra:Vinod Pustak Mandir.
5) Sidhu, K.S. : "Teaching of Mathematics", New Delhi sterling Publication, 1990
6) Sobel,Max A.: "Teaching of general Mathematics", Englewood cliffs, New Jersery:Prentice Hall,1967.

7) Kumar, Sudhir and Ratnalikar, D.N. : “Teaching of Mathematics”, New Delhi: Anmol Publications, 2003
8) Swami Bidyaranya Swami and Singh Avadesh: “History of Hindu Mathemaitcs”, a source Book, Lahore: Motial Banarsidas, 1935
9) Boyer, Carl B.,H. “History of Mathematics” New York: John Wiley, 1968
10) Dalal,D.C.:”Teaching of Mathematics”, Ludhiana: Vijay Publications
11) Mangal,S.K.:”Teaching of Mathematics” Ludhiana: Parkash Book Depot
12) Datta,B. and Singh, A.N.: “History of Hindu Mathematics”, 1933
13) Crandall, I.R. & Pomerance, C : “*Prime Numbers: A Computational Perspective”*, Springer, NY, 2001: 1.2.4
14) Kulsheshetha, A.K: “Teaching of Mathematics.” Meerut, R. Lal Book Depot.

Lesson No. 1.4 **Srinivasa Ramanujan**

1.4.1 Objectives

1.4.2 Introduction: Srinivasa Ramanujan

1.4.3 Contribution towards mathematics

1.4.4 Introduction of Aryabhata

1.4.4.1 Contribution towards mathematics

1.4.5 Introduction of Bhaskarcharya

1.4.5.1 Contribution towards mathematics

1.4.6 Summary

1.4.7 Self Check Exercise

1.4.8 Suggested questions

1.4.9 Suggested books and web sources

1.4.1 Objectives

After reading this lesson students will be able to:

1. Describe the life sketch of Ramanujan's life.
2. Explain the relationship between Ramanujan and Hardy.
3. Discuss the contributions of Ramanujan's in mathematics.
4. Describe the life sketch of Aryabhata's and Bhaskarcharya's life.
5. Explain the contributions of Aryabhata in mathematics.

1.5. Discuss the contributions of Bhaskarcharya in mathematics.

1.4.2 Introduction: Srinivasa Ramanujan

Ramanujan was born on 22 December 1887 in Erode, Madras Presidency , at the residence of his maternal grandparents. His father, K. Srinivasa Iyengar, worked as a clerk in a sari shop and hailed from the district of Thanjavur. His mother, Komalatammal, was a housewife and also sang at a local temple. They lived in Sarangapani Street in a traditional home in the town of Kumbakonam. The family home is now a museum. On 1 October 1892, Ramanujan was enrolled at the local school.

He did not like school, and he tried to avoid attending. His family enlisted a local constable to make sure he attended school. Since Ramanujan's father was at work most of the day, his mother took care of him as a child. He had a close relationship with her. From her, he learned about tradition and puranas. He learned to sing religious songs, to attend pujas at the temple and particular eating habits – all of which are part of Brahmin culture. Just before the age of 10, in November 1897, he passed his primary examinations in English, Tamil, geography and arithmetic. With his scores, he stood first in the district .That year, Ramanujan entered Town Higher Secondary School where he encountered formal mathematics for the first time.

He was later lent a book on advanced trigonometry written by S. L. Loney. He completely mastered this book by the age of 13 and discovered sophisticated theorems on his own. By 14, he was receiving merit certificates and academic awards which continued throughout his school career and also assisted the school in the logistics of assigning its 1200 students (each with their own needs) to its 35 odd teachers. He completed mathematical exams in half the allotted time, and showed a familiarity with geometry and infinite series. Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quartic.

In 1903 when he was 16, Ramanujan obtained a book by G. S. Carr. The book was titled *A Synopsis of Elementary Results in Pure and Applied Mathematics* and was a collection of 5000 theorems. Ramanujan reportedly studied the contents of the book in detail .The book is generally acknowledged as a key element in awakening the genius of Ramanujan .The next year, he had independently developed and investigated the Bernoulli numbers and had calculated the Euler–Mascheroni constant up to 15 decimal places.

When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. Ranganatha Rao prize for mathematics by the school's headmaster, Krishnaswami Iyer. Iyer introduced Ramanujan as an outstanding student who deserved scores higher than the maximum possible marks.He received a scholarship to study at Government Arts College, Kumbakonam. However, Ramanujan was so intent on studying mathematics that he could not focus on any other subjects and failed most of them, losing his scholarship in the process. In August 1905, he ran away from home, heading towards Visakhapatnam and stayed in Rajahmundry for about a month. He later enrolled at Pachaiyappa's College in Madras. He again excelled in mathematics but performed poorly in other subjects such as physiology. Ramanujan failed his Fellow of Arts exam in December 1906 and again a year later. Without a degree, he left college and continued to pursue independent research in mathematics.

Ramaswamy Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras. Some of these friends looked at his work and gave him letters of introduction to R. Ramachandra Rao, the district collector for Nellore and the secretary of the Indian Mathematical Society. Ramachandra Rao was impressed by Ramanujan's research.

When Rao asked him what he wanted, Ramanujan replied that he needed some work and financial support. Rao consented and sent him to Madras. He continued his mathematical research with Rao's financial aid taking care of his daily needs. Ramanujan, with the help of Ramaswamy Aiyer, had his work published in the *Journal of the Indian Mathematical Society*.

Ramanujan wrote his first formal paper for the *Journal* on the properties of Bernoulli numbers. .Ramanujan later wrote another paper and also continued to provide problems in the *Journal*.

After he saw Ramanujan's theorems on continued fractions on the last page of the manuscripts, Hardy commented that "they [theorems] defeated me completely; I had never seen anything in the least like them before". He figured that Ramanujan's theorems "must be true, because, if they were not true, no one would have the imagination to invent them". Hardy asked a colleague, J. E. Littlewood, to take a look at the papers. Littlewood was amazed by the mathematical genius of Ramanujan. After discussing the papers with Littlewood, Hardy concluded that the letters were "certainly the most remarkable I have received" and commented that Ramanujan was "a mathematician of the highest quality, a man of altogether exceptional originality and power". One colleague, E. H. Neville, later commented that "not one [theorem] could have been set in the most advanced mathematical examination in the world".

Ramanujan continued to submit papers to the *Journal of the Indian Mathematical Society*. Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood and published a part of his findings there. Hardy and Ramanujan had highly contrasting personalities. Their collaboration was a clash of different cultures, beliefs and working styles. Hardy was an atheist and an apostle of proof and mathematical rigour, whereas Ramanujan was a deeply religious man and relied very strongly on his intuition. While in England, Hardy tried his best to fill the gaps in Ramanujan's education without interrupting his spell of inspiration.

Ramanujan was awarded a Bachelor of Science degree by research (this degree was later renamed PhD) in March 1916 for his work on highly composite numbers, the first part of which was published as a paper in the *Proceedings of the London Mathematical Society*. The paper was over 50 pages with different properties of such numbers proven. Hardy remarked that this was one of the most unusual papers seen in mathematical research at that time and that Ramanujan showed extraordinary ingenuity in handling it. On 6 December 1917, he was elected to the London Mathematical Society. He became a Fellow of the Royal Society in 1918, becoming the second Indian to do so, following Ardaseer Cursetjee in 1841, and he was one of the youngest Fellows in the history of the Royal Society. He was elected "for his investigation in Elliptic functions and the Theory of Numbers." On 13 October 1918, he became the first Indian to be elected a Fellow of Trinity College, Cambridge.

Ramanujan returned to Kumbakonam, Madras Presidency in 1919 and died soon thereafter at the age of 32 in 1920. His widow, S. Janaki Ammal, moved to Mumbai, but returned to Chennai (formerly Madras) in 1950, where she lived until her death at age 95 in 1994

1.4.3 Contribution in Mathematics

1. Ramanujam made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions and infinite 1900 he began to work on his own on mathematics summing geometric and arithmetic series.
2. He worked on **divergent series**. He sent 120 theorems on imply divisibility properties of the partition function.
3 He gave a meaning to **eulerian second integral** for all values of n (negative, positive and fractional). He proved that the integral of $x^{n-1} e^{-7} = \Upsilon$ (gamma) is true for all values of gamma.
4 **Goldbach's conjecture:** Goldbach's conjecture is one of the important illustrations of ramanujan contribution towards the proof of the conjecture. The statement is every even integer greater that two is the sum of two primes, that is, 6=3+3 : Ramanujan and his associates had shown that every large integer could be written as the sum of at most four (Example: 43=2+5+17+19).
5 **Partition of whole numbers**: Partition of whole numbers is another similar problem that captured ramanujan attention. Subsequently ramanujan developed a formula for the partition of any number, which can be made to

yield the required result by a series of successive approximation. Example 3=3+0=1+2=1+1+1;

6 **Numbers:** Ramanujan studied the highly composite numbers also which are recognized as the opposite of prime numbers. He studies their structure, distribution and special forms.

7 **Fermat Theorem:** He also did considerable work on the unresolved Fermat theorem, which states that a prime number of the form 4m+1 is the sum of two squares.

8 **Ramanujan number:** 1729 is a famous ramanujan number. It is the smaller number which can be expressed as the sum of two cubes in two different ways- $1729 = 1^3 + 12^3 = 9^3 + 10^3$

9 **Cubic Equations and Quadratic Equation:** Ramanujam was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quadratic. The following year, not knowing that the quintic could not be solved by radicals, he tried (and of course failed) to solve the quintic.

10 **Euler's constant :** By 1904 Ramanujam had began to undertake deep research. He investigated the series (1/n) and calculated *Euler's constant* to 15 decimal places.

11 **Hypo geometric series:** He worked hypo geometric series, and investigated relations between integrals and series. He was to discover later that he had been studying elliptic functions. Ramanujan's own works on partial sums and products of hyper-geometric series have led to major development in the topic.

12 **Journal *of the Indian mathematical society*:** Ramanujan continued to develop his mathematical ideas and began to pose problems and solve problems in the journal *of the Indian mathematical society.* He developed relations between elliptic modular equations in 1910.

13 **Bernoulli numbers:** He published a brilliant research paper on Bernoulli numbers in 1911 in the journal of the Indian mathematical society and gained recognition for his work. Despite his lack of a university education, he was becoming well known in the madras area as a mathematical genius. He began to study the Bernoulli numbers, although this was entirely his own independent discovery.

14 Ramanujan worked out the Riemann series, the elliptic integrals hyper geometric series and functions equations of the zeta functions on the other hand he had only a vague idea of what constitutes a mathematical proof. Despite many brilliant results, some of his theorems on prime numbers were completely wrong.

15 Ramanujan independently discovered results of ***gauss, Kummar and others on hyper-geometric series.***

16 Perhaps has most famous work was on the number p(n) for small numbers n, and ramaujan used this numerical data to conjecture some remarkable properties some of which he proved using elliptic functions. others were only proved after Ramanujan's death. In a joint paper with hardly, ramanujan gave an asymptotic formulas for p(n). It had the remarkable property that it

appeared to give the correct value of p(n), and this was laterproved by Rademacher.

17 Ramanujan discovered a number of remarkable identities that imply divisibility properties of the partition function. He also produced quite a number of results in definite integrals in the form of general formulate.

18. His chief contribution in mathematics lies mainly in analysis, game theory and infinite series. He made in depth analysis in order to solve various mathematical problems by bringing to light new and novel ideas that gave impetus to progress of game theory. Such was his mathematical genius that he discovered his own theorems. It was because of his keen insight and natural intelligence that he came up with infinite series for π

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum^{\infty} \frac{(4k)!(1103 + 2}{(k!)43964}$$

This series made up the basis of certain algorithms that are used today.

1.4.4 Introduction of Aryabhat

Aryabhata was born in 476CE and died in 550CE. His main interests were mathematics and astronomy. His works include the Aryabhatiya and the Arya-siddhanta.

Aryabhata provides no information about his place of birth. The only information comes from Bhāskara I, who describes Aryabhata as *āśmakīya*, "one belonging to the *aśmaka* country." During the Buddha's time, a branch of the Aśmaka people settled in the region between the Narmada and Godavari rivers in central India; Aryabhata is believed to have been born there.

It has been claimed that the *aśmaka* (Sanskrit for "stone") where Aryabhata originated may be the present day Kodungallur which was the historical capital city of *Thiruvanchikkulam* of ancient Kerala.[1] This is based on the belief that Koṭuṅṅallūr was earlier known as Koṭum-Kal-l-ūr ("city of hard stones"); however, old records show that the city was actually Koṭum-kol-ūr ("city of strict governance"). Similarly, the fact that several commentaries on the Aryabhatiya have come from Kerala has been used to suggest that it was Aryabhata's main place of life and activity; however, many commentaries have come from outside Kerala, and the Aryasiddhanta was completely unknown in Kerala. K. Chandra Hari has argued for the Kerala hypothesis on the basis of astronomical evidence.

Aryabhata mentions "Lanka" on several occasions in the *Aryabhatiya*, but his "Lanka" is an abstraction, standing for a point on the equator at the same longitude as his Ujjayini.

Aryabhata went to Kusumapura for advanced studies and lived there for some time. Both Hindu and Buddhist tradition, as well as Bhāskara I (CE 629), identify Kusumapura as Pāṭaliputra, modern Patna. A verse mentions that Aryabhata was the head of an institution (*kulapa*) at Kusumapura, and, because the university of Nalanda was in Pataliputra at the time and had an astronomical observatory, it is speculated that Aryabhata might have been the head of the Nalanda university as well. Aryabhata is also reputed to have set up an observatory at the Sun temple in Taregana, Bihar.

His major work, *Aryabhatiya* covers arithmetic, algebra, plane trigonometry, and spherical trigonometry. It also contains continued fractions, quadratic equations, sums-of-power series, and a table of sines.

The *Arya-siddhanta*, a lost work on astronomical computations, is known through the writings of Aryabhata's contemporary, Varahamihira, and later mathematicians and commentators, including Brahmagupta and Bhaskara I. This work appears to be based on the older Surya Siddhanta and uses the midnight-day reckoning, as opposed to sunrise in *Aryabhatiya*. It also contained a description of several astronomical instruments: the gnomon (*shanku-yantra*), a shadow instrument (*chhAyA-yantra*), possibly angle-measuring devices, semicircular and circular (*dhanur-yantra* / *chakra-yantra*), a cylindrical stick *yasti-yantra*, an umbrella-shaped device called the *chhatra-yantra*, and water clocks of at least two types, bow-shaped and cylindrical.

. Aryabhatiya is divided into 4 chapters called Pada (section)

- **Pada 1 – GitikaPada** – 13 stanzas of basis definition of important astronomical parameters and tables.
- **Pada 2 – Ganita Pada** – 33 stanzas deals with mathematics. The topics are geometrical figures with their properties and mensurations, series, linear and quadratic equations, methods for extracting the square roots, the cube roots etc.
- **Pada 3 – Kalakriya Pada** – 25 stanzas deals with the true position of sun, moon and planets.

- **Pada 4 – Gola Pada** – 50 stanzas deals with the motion of sun, moon and planets on the celestial sphere.

1.4.4.1 CONTRIBUTION IN MATHEMATICS:

NUMBER NOTATION

- **Numerical values:** he made a notation system in which digits are denoted with the help of alphabet numerals e.g., 1 = ka, 2 = Kha, etc.

Aryabhatta assigned numerical values to the 33 consonants of the Indian alphabet to represent 1,2,3…25,30,40,50,60,70,80,90,100.

- ***Notation system:*** He invented a notation system consisting of alphabet numerals Digits were denoted by alphabet numerals. In this system devanagiri script contain varga letters (consonants) and avarga letters (vowels).1-25 are denoted by 1st 25 varga letters.
- ***Place-value:*** Aryabhatta was familiar with the place-value system.
- He knew numeral symbols and the sign for zero
- ***Square root & cube root:*** His calculations on square root and cube root would not have been possible without the knowledge of place values system and zero. He has given methods of extracting square root cube root along with their explanation.
- ***Interest:*** He formulated for the first time in India the formula for interest, time and other related ones, in the problems of interest.

ALGEBRA

- ***Integer solutions:*** Aryabhatta was the first one to explore integer solutions to the equations of the form by =ax+c and by =ax-c, where a,b,c are integers. He used kuttuka method to solve problems.
- ***Indeterminate equations:*** He gave general solutions to linear indeterminate equations ax+by+c= 0 by the method of continued fraction.
- ***Identities:*** He had dealt with identities like $(a+b)^2=a^2+2ab+b^2$ and $ab=\{(a+b)^2-(a^2-b^2)\}/2$
- He has given the following formula in aryabhatia
  - $1^2+2^2+3^2+\text{---------}+n^2=n(n+1)(2n+1)/6$

    $1^3+2^3+3^3+\text{---------}+n^3 = (1+2+3+\text{------------}+)^2= \{n^2(n+1)^2\}/4$

- ***Algebraic quantities:*** He has given the method of addition, subtraction, multiplication of simple and compound algebraic quantities
- ***Arithmetic series:*** He was given a formula for summing up of the arithmetic series after the P^{th} term The rule is S= n[a+{(n-1)/2+p} d]

  S=(a+1) n/2

GEOMETRY

- ***Discover the Π Value :*** The credit for discovering the exact values Π may be ascribed to the celebrated mathematician Aryabhatta.

Rule: Add 4 to 100, multiply by 8, add 62000. The result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given.

This gives Π =62832/20000=3.1411.5. Which is an accurate value of Π. Aryabhatta discovered this value independently and also realized that Π is an irrational number

- ***Pythagorean Theorem:*** The Pythagorean theorem is stated as follows in his work “the square of the Bhuja (base) plus the square of the koti (perpendicular) is the square of the Karna”

  (Buja and koti are the sides of a right-angled triangle. The Karna is the hypotenuse)

- ***Circle Theorem:*** He has postulated a theorem relating to circle as follows “In a circle the product of two Saras is the square of the half chord of the two arcs” i.e. $a*b=c^2$ where c is half the chord and the saras or arrows are the segments of a diameter which bisect any chord.
- ***Formula:*** Aryabhatta gives formulae for the areas of a triangle, square, rectangle, rhombus, circle etc.

TRIGONOMETRY

- ***Sine Table:*** Aryabhatta gave a table of sines for calculating the approximate values at intervals of 90/24 = 3 45’. This was done using the formula for
  - sin (n+1)x - sin nx in terms of sin nx and sin (n-1) x.
- ***Versine:*** He introduced the versine (versin = 1-cosine) into trigonometry.

ASTRONOMY

- ***Earth:*** Aryabhatta gave the circumference of the earth as 4 967 yojanas and its diameter as 1 5811/24 yojanas. Since1 yojana =5miles this gives the circumference as 24,835 miles, which is an excellent approximation to the currently accepted value of 24,902 miles.
- He believes that the orbits of the planets are ellipses. He correctly explains the caused of eclipses of the Sun and the Moon.
- **Length of year:** His value for the length of the year at 365 days 6 hours 12 minutes 30 seconds is an overestimate since the true value is less than 365 days and 6 hours.

1.4.5 Introduction of Bhaskarcharya

Bhaskaracharya otherwise known as Bhaskara II was one of the most powerful and creative mathematicians of ancient India. He was also known as Bhaskara the Learned. He was born in 1114 AD in Vijayapura. His father, Mahesvara, himself was a famous astrologer. In many ways Bhaskaracharya represents the peak of mathematical knowledge of 12th century. He was the head of the astronomical observatory at Ujjain.

His main work *Siddhānta Shiromani*, (Sanskrit for "Crown of Treatises") is divided into four parts called *Lilāvatī*, *Bījagaṇita*, *Grahagaṇita* and *Golādhyāya*, which are also sometimes considered four independent works. These four sections deal with arithmetic, algebra, mathematics of the planets, and spheres respectively. He also wrote another treatise named Karaṇa Kautūhala.

Bhāskara's work on calculus predates Newton and Leibniz by over half a millennium. He is particularly known in the discovery of the principles of differential calculus and its application to astronomical problems and computations. While Newton and Leibniz have been credited with differential and integral calculus, there is strong evidence to suggest that Bhāskara was a pioneer in some of the principles of differential calculus. He was perhaps the first to conceive the differential coefficient and differential calculus.

he composed the Siddhānta Śiromaṇī when he was 36 years old. He also wrote another work called the *Karaṇa-kutūhala* when he was 69 (in 1183). His works show the influence of Brahmagupta, Sridhara, Mahāvīra, Padmanābha and other predecessors.

Lilavathi contains 13 chapters and covers topics such as Définition, Mathematical terms, Interest, Arithmetical progression and Geometrical progression , Plane geometry , Solid geometry , Shadow of the gnomon , Kuttaka and combinations.

Bijaganita contains 12 chapters and covers topics like as Positive and negative numbers, Zero , Surds, the kuttaka , Indeterminate quadratic equation with more than one unknown , Quadratic equation with more than one unknown , Operations with products of several unknown.

Siddhantasironmani is a mathematical astronomy book compiled in two parts. *First part contains twelve chapters dealing with topics such as Longitudes of the planets, True longitudes of the planets, 3 problems of decimal rotation , syzygies, Lunar eclipse , Solar eclipse, Latitudes of the planets , Rising and setting , Moon's crescent , Conjunction of the planets , The pates of sun and moon*

Second part of siddhantasiromani contains 13 chapters on the sphere. The topics such as: Praise of study of the sphere , Nature of the sphere , Cosmography and geography , Planetary mean motion , Eccentric epicyclic model ,The armillary sphere , Spherical trigonometry , Ellipse calculations , First visibilities of planets , Calculating the lunar crescents , Astronomical instruments , Problems of astronomical calculations.

1.4.5.1 CONTRIBUTION IN MATHEMATICS:

Negative Numbers:

- Bhaskaracharya was known for his treatment of negative numbers with he considered as debts or losses, and also for his treatise on arithmetic and measurement.
- Bhaskaracharya also handled efficiently arithmetic involving negative numbers.
- In Bijaganita placing a dot above them denotes negative numbers.

Infinity & Zero:

- He for the first time brought the idea of infinity while dividing a number by zero.

Zero rules:

- He was sound in addition, subtraction and multiplication involving zero but realized that there were problems with Brahmagupta's idea of dividing by zero.
  - A + 0 = A
  - A – 0 = A
  - A x 0 = 0
- He understood about zero and negative numbers and he knew that x^2 = 9 had two solutions.

Progression:

- He was aware of arithmetical and geometrical progression and explains examples.

Sphere:

- He found formula for finding the area and volume of sphere given below:

Area of sphere = 4 x area of a circle.

Volume of a sphere = area of a sphere x 1/6 of its diameter.

Trigonometry:

- He seems more interested in trigonometry. Among the many interesting results given by bhaskaracharya are:
- sin (a + b) = sin a cos b + cos a sin b
- And sin (a - b) = sin a cos b - cos a sin b.

***Lilavati*:**

- Bhaskaracharya gave two methods of multiplication in Lilavati.
- It is argued that zero used by bhaskaracharya, in his rule (a.0)/0 = a given in Lilavathi, is equivalent to the modern concept of a non-zero "infinitesimal".

Other works:

- He has used the kuttaka method of solving indeterminate equations.
- He had explained the concepts of permutation combination with examples.

- In differential calculus he was the first mathematician who presented examples related to differential coefficient.
- He originated the fundamentals of Rolle's theorem.
- He knew about inverse proportions and rule of the three

. Bhaskaracharya's innumerable contribution had earned him an outstanding position among the ancient Hindu mathematicians. He dies in Ujjain in 1185.

Short in Text Questions

1. Explain Contribution of Mathematician Ramanujan.
 ……………………………………………………………………………………………
 ……………………………………………………………………………………………
2. Discuss in Contribution of Aryabhatt
 ……………………………………………………………………………………………
 ……………………………………………………………………………………………

1.4.6 Summary

In this lesson we discussed about the Mathematics Srinvasa Ramanujan Aryabhath and Bhaskarcharaya and the Conlibution in Mathematics. Srinivasa Ramanujan was a world renowned Indian Mathematician who had mathematical manipulative abilities the excelled in number theory and modular function.

1.4.7 Self Check Exercise

Fill in the blank

1. Ramanujan was born in……………………..
2. Ramanujan was awarded ………………. prize for mathematics.
3. Aryabhatiya in divided into 4 chapter called……………………..

1.4.8 Suggested Questions:

1. Discuss the life sketch of Aryabhata.
2. Explain the life history of Bhaskarcharaya.
3. What are the main contributions of Aryabhata's and Bhaskarcharya's in mathematics?
4. Discuss the relationship of Ramanujan with English mathematicians.
5. Explain the life history of Ramanujan.
6. What are the main contributions of Ramanujan's in mathematics?

1.4.9 Suggested books:

1. Mangal,S.K.: “Teaching of Mathematics”,New Delhi:Arya Book Depot,2008….

2.Sidhu,K.S.: “Teaching of Mathematics”,New Delhi sterlimg Publication,1990

3. Mangal,S.K.: “Teaching of Mathematics”,Ludhiana Parkash Book Depot

4. Kulsheshetha,A.K: “Teaching of Mathematics”,Meerut,R.Lal Book Depot

1. Mangal,S.K.: “Teaching of Mathematics”,New Delhi:Arya Book Depot,2008….

2.Sidhu,K.S.: “Teaching of Mathematics”,New Delhi sterlimg Publication,1990

3. Mangal,S.K.: “Teaching of Mathematics”,Ludhiana Parkash Book Depot

4. Kulsheshetha,A.K: “Teaching of Mathematics”,Meerut,R.Lal Book Depot

B.ed (Part-I) Semester –I **PAPER IV & V (Opt.-ix)**

Teaching of Mathematics

Euclid and Pythagoras

Lesson No. 1.5

1.5.1 Objectives

1.5.2 Introduction of Euclid

1.5.3 Contribution towards mathematics

1.5.4 Introduction and contribution of Pythagoras

1.5.5 Summary

1.5.6 Self Check Exercise

1.5.7 Suggested questions

1.5.8 Suggested books and web sources

1.5.1 Objectives

After reading this lesson students will be able to:

i. Describe the life sketch of Euclid's and Pythagoras's life.

ii. Explain the contributions of Euclid in mathematics.

iii. Discuss the contributions of Pythagoras in mathematics.

1.5.2 Introduction of Fuclid

He was born in Mid-4th century BC and died in Mid-3rd century BC.

Euclid (300 BC), sometimes called **Euclid of Alexandria**, was a Greek mathematician, often referred to as the "Father of Geometry". He was active in Alexandria during the reign of Ptolemy I (323–283 BC). His *Elements* is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. In the *Elements*, Euclid deduced the principles of what is now called Euclidean geometry from a small set of axioms. Euclid also wrote works on perspective, conic sections, spherical geometry, number theory and rigor.

Very few original references to Euclid survive, so little is known about his life. The date, place and circumstances of both his birth and death are unknown and may only be estimated roughly. He is rarely mentioned by name by other Greek mathematicians from Archimedes onward, who usually call him "ὁ στοιχειώτης" ("the author of Elements"). The few historical references to Euclid were written centuries after he lived, by Proclus c. 450 AD and Pappus of Alexandria c. 320 AD.

Proclus introduces Euclid only briefly in his *Commentary on the Elements*. According to Proclus, Euclid belonged to Plato's "persuasion" and brought together the *Elements*, drawing on prior work by several pupils of Plato (particularly Eudoxus of Cnidus, Theaetetus and Philip of Opus.) Proclus believes that Euclid is not much younger than these, and that he must have lived during the time of Ptolemy I because he was mentioned by Archimedes (287–212 BC). Although the apparent citation of Euclid by Archimedes has been judged to be an interpolation by later editors of his works, it is still believed that Euclid wrote his works before those of Archimedes.

Proclus later retells a story that, when Ptolemy I asked if there was a shorter path to learning geometry than Euclid's *Elements*, "Euclid replied there is no royal road to geometry." This anecdote is questionable since it is similar to a story told about Menaechmus and Alexander the Great.

In the only other key reference to Euclid, Pappus briefly mentioned in the fourth century that Apollonius "spent a very long time with the pupils of Euclid at Alexandria, and it was thus that he acquired such a scientific habit of thought" c. 247–222 BC.

A detailed biography of Euclid is given by Arabian authors, mentioning, for example, a birth town of Tyre. This biography is generally believed to be completely fictitious.

Euclid gathered up all of the knowledge developed in Greek mathematics at that time and created his great work, a book called 'The Elements' (c300 BCE). This treatise is unequaled in the history of science and could safely lay claim to being the most influential non-religious book of all time.

Euclid probably attended Plato's academy in Athens before moving to Alexandria, in Egypt. At this time, the city had a huge library and the ready availability of papyrus made it the center for books, the major reasons why great minds such as Heron of Alexandria and Euclid based themselves there.

1.5.3 Contribution in Mathematics

Euclid's Elements

Euclid's great work consisted of thirteen books
covering a vast body of mathematical knowledge,
spanning arithmetic, geometry and number theory. The books are organized by subjects, covering every area of mathematics developed by the Greeks:

- Books I - IV, and Book VI: Plane Geometry
- Books XI - XIII: Solid Geometry
- Books V and X: Magnitudes and Ratios
- Books VII - IX: Whole Numbers

The basic structure of the elements begins with Euclid establishing axioms, the starting point from which he developed 465 propositions, progressing from his first established principles to the unknown in a series of steps, a process that he called the 'Synthetic Approach.' He looked at mathematics as a whole, but was concentrated on geometry and that particular discipline formed the basis of his work.

Euclid's Axioms

Euclid based his approach upon 10 axioms, statements that could be accepted as truths. He called these axioms his 'postulates' and divided them into two groups of five, the first set common to all mathematics, the second specific to geometry. Some of these postulates seem to be self-explanatory to us, but

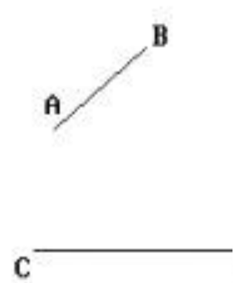
Euclid operated upon the principle that no axiom could be accepted without proof.

Euclid's First Group of Postulates - the Common Notions:

1. Things which are equal to the same thing are also equal to each other
2. If equals are added to equals, the results are equal
3. If equals are subtracted from equals, the remainders are equal
4. Things that coincide with each other are equal to each other
5. The whole is greater than the part

The remaining five postulates were related specifically to geometry:

1. A straight line can be drawn between any two points.
2. Any finite straight line can be extended indefinitely in a straight line.
3. For any line segment, it is possible to draw a circle using the segment as the radius and one end point as the center.
4. All right angles are congruent (the same).
5. If a straight line falling across two other straight lines results in the sum of the angles on the same side less than two right angles, then the two straight lines, if extended indefinitely, meet on the same side as the side where the angle sums are less than two right angles.



From Euclids Elements Book I, Proposition 3(Creative Commons)

Euclid felt that anybody who could read and understand words could understand his notions and postulates but, to make sure, he included 23 definitions of common words, such as 'point' and 'line', to ensure that there could be no semantic errors. From this basis, he built his entire theory of plane geometry, which has shaped mathematics, science and philosophy for centuries. He proved that it is an impossibility to find the 'largest prime number,' because taking the largest known prime number and adding one to the product of all previous primes and the largest prime will give you another, larger prime number.

Although many of the results in *Elements* originated with earlier mathematicians, one of Euclid's accomplishments was to present them in a single, logically coherent framework, making it easy to use and easy to reference, including a system of rigorous mathematical proofs that remains the basis of mathematics 23 centuries later.

There is no mention of Euclid in the earliest remaining copies of the *Elements*, and most of the copies say they are "from the edition of Theon" or the "lectures of Theon",[] while the text considered to be primary, held by the Vatican, mentions no author. The only reference that historians rely on of Euclid having written the *Elements* was from Proclus, who briefly in his *Commentary on the Elements* ascribes Euclid as its author.

Although best known for its geometric results, the *Elements* also includes number theory. It considers the connection between perfect numbers and Mersenne primes (known as the Euclid–Euler theorem), the infinitude of prime numbers, Euclid's lemma on factorization (which leads to the fundamental theorem of arithmetic on uniqueness of prime factorizations), and the Euclidean algorithm for finding the greatest common divisor of two numbers.

The geometrical system described in the *Elements* was long known simply as *geometry*, and was considered to be the only geometry possible. Today, however, that system is often referred to as *Euclidean geometry* to distinguish it from other so-called *non-Euclidean geometries* that mathematicians discovered in the 19th century.

Other works

In addition to the *Elements*, at least five works of Euclid have survived to the present day. They follow the same logical structure as *Elements*, with definitions and proved propositions.

- *Data* deals with the nature and implications of "given" information in geometrical problems; the subject matter is closely related to the first four books of the *Elements*.

- *On Divisions of Figures*, which survives only partially in Arabic translation, concerns the division of geometrical figures into two or more equal parts or into parts in given ratios. It is similar to a third-century AD work by Heron of Alexandria.
- *Catoptrics*, which concerns the mathematical theory of mirrors, particularly the images formed in plane and spherical concave mirrors. The attribution is held to be anachronistic however by J J O'Connor and E F Robertson who name Theon of Alexandria as a more likely author.[18]
- *Phaenomena*, a treatise on spherical astronomy, survives in Greek; it is quite similar to *On the Moving Sphere* by Autolycus of Pitane, who flourished around 310 BC.
- *Optics* is the earliest surviving Greek treatise on perspective. In its definitions Euclid follows the Platonic tradition that vision is caused by discrete rays which emanate from the eye. One important definition is the fourth: "Things seen under a greater angle appear greater, and those under a lesser angle less, while those under equal angles appear equal." In the 36 propositions that follow, Euclid relates the apparent size of an object to its distance from the eye and investigates the apparent shapes of cylinders and cones when viewed from different angles. Proposition 45 is interesting, proving that for any two unequal magnitudes, there is a point from which the two appear equal. Pappus believed these results to be important in astronomy and included Euclid's *Optics*, along with his *Phaenomena*, in the *Little Astronomy*, a compendium of smaller works to be studied before the *Syntaxis* (*Almagest*) of Claudius Ptolemy.

1.5.4 Introduction and Contribution

Pythagoras

Pythagoras of Samos (Greek: "Pythagoras the Samian", or simply in Ionian Greek; c. 570 – c. 495 BC) was an Ionian Greek philosopher, mathematician, and has been credited as the founder of the movement called Pythagoreanism. Most of the information about Pythagoras was written down centuries after he lived, so very little reliable information is known about him. He was born on the island of Samos, and traveled, visiting Egypt and Greece, and maybe India, and in 520 BC returned to Samos. Around 530 BC, he moved to Croton, in Magna Graecia, and there established some kind of school or guild.

Pythagoras made influential contributions to philosophy and religion in the late 6th century BC. He is often revered as a great mathematician and scientist and is best known for the Pythagorean theorem which bears his name.

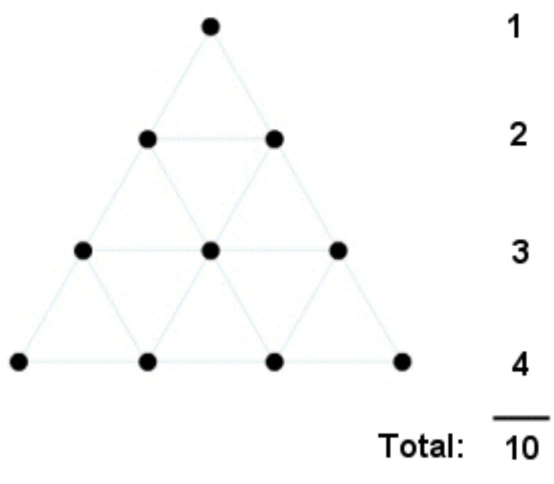
His father was Mnesarchus and his mother was Pythais. Pythagoras spent his early years in Samos. There is little known about his child hood and all physical descriptions of Pythagoras are said to be fictitious except for the vivid birthmark on his thigh. It is believed that he had two brothers and some believe there were three.

Pythagoras was extremely well educated. There were three philosophers that influenced him while he was young. One of the most important of these man was a man named Pherekydes. The philosopher that introduced Pythagoras to mathematical ideas was Thales , who lived in Miletus. It was because of Thales that Pythagoras became interested in math, astronomy and cosmology

The school he established at Croton in southern Italy around 530 BC. **Pythagoras's** school was “All is number” or “God is number”, and the Pythagoreans effectively practised a kind of numerology or number-worship, and considered each number to have its own character and meaning. For example, the number one was the generator of all numbers; two represented opinion; three, harmony; four, justice; five, marriage; six, creation; seven, the seven planets or “wandering stars”; etc. Odd numbers were thought of as female and even numbers as male.

The holiest number of all was "tetractys" or ten, a triangular number composed of the sum of one, two, three and four. It is a great tribute to the Pythagoreans' intellectual achievements that they deduced the special place of the number 10 from an abstract mathematical argument rather than from something as mundane as counting the fingers on two hands.

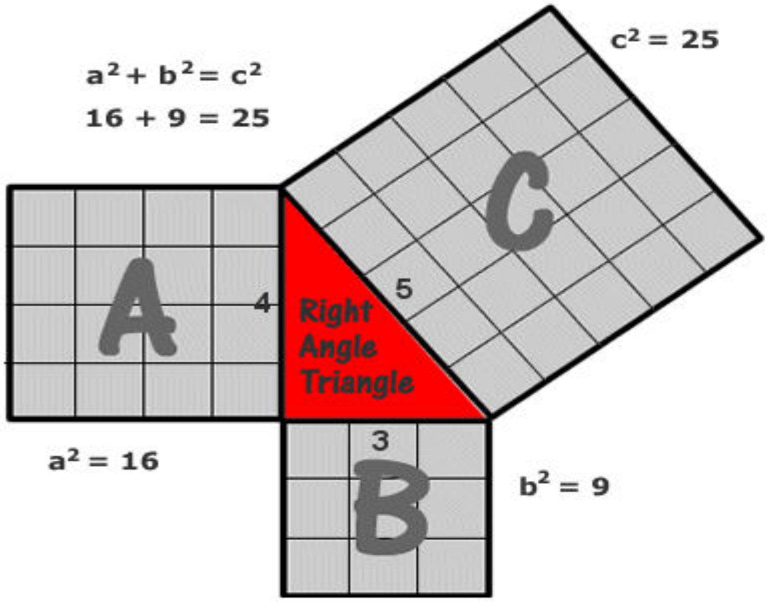
The tetractys, an equilateral triangular figure consisting of 10 points arranged in four rows of 1, 2, 3 and 4, was both a mathematical idea and a metaphysical symbol for the Pythagoreans.



However, Pythagoras and his school - as well as a handful of other mathematicians of ancient Greece - was largely responsible for introducing a more rigorous mathematics than what had gone before, building from first principles using axioms and logic. Before Pythagoras, for example, geometry had been merely a collection of rules derived by empirical measurement. Pythagoras discovered that a complete system of mathematics could be constructed, where geometric elements corresponded with numbers, and where integers and their ratios were all that was necessary to establish an entire system of logic and truth.

He is mainly remembered for what has become known as Pythagoras' Theorem (or the Pythagorean Theorem): that, for any right-angled triangle, the square of the length of the hypotenuse (the longest side, opposite the right angle) is equal to the sum of the square of the other two sides (or "legs"). Written as an equation: $a^2 + b^2 = c^2$. What Pythagoras and his followers did not realize is that this also works for any shape: thus, the area of a pentagon on the hypotenuse is equal to the sum of the pentagons on the other two sides, as it does for a semi-circle or any other regular (or even irregular shape).

The simplest and most commonly quoted example of a Pythagorean triangle is one with sides of 3, 4 and 5 units ($3^2 + 4^2 = 5^2$, as can be seen by drawing a grid of unit squares on each side as in the diagram at right), but there are a potentially infinite number of other integer "Pythagorean triples", starting with (5, 12 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), (9, 40, 41), etc. It should be noted, however that (6, 8, 10) is not what is known as a "primitive" Pythagorean triple, because it is just a multiple of (3, 4, 5).



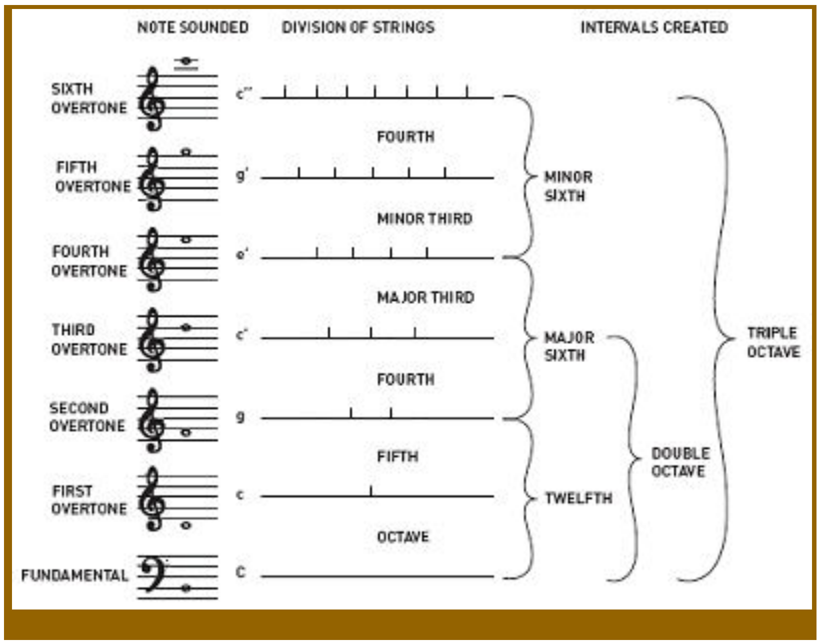
It soon became apparent, though, that non-integer solutions were also possible, so that an isosceles triangle with sides 1, 1 and √2, for example, also has a right angle, as the Babylonians had discovered centuries earlier. However, when Pythagoras's student Hippasus tried to calculate the value of √2, he found that it was not possible to express it as a fraction, thereby indicating the potential existence of a whole new world of numbers, the irrational numbers (numbers that can not be expressed as simple fractions of integers). This discovery rather shattered the elegant mathematical world built up by Pythagoras and his followers, and the existence of a number that could not be expressed as the ratio of two of God's creations (which is how they thought of the integers) jeopardized the cult's entire belief system.

Poor Hippasus was apparently drowned by the secretive Pythagoreans for broadcasting this important discovery to the outside world. But the replacement of the idea of the divinity of the integers by the richer concept of the continuum, was an essential development in mathematics. It marked the real birth of Greek geometry, which deals with lines and planes and angles, all of which are continuous and not discrete.

Among his other achievements in geometry, Pythagoras (or at least his followers, the Pythagoreans) also realized that the sum of the angles of a triangle is equal to two right angles (180°), and probably also the generalization which states that the sum of the interior angles of a polygon with n sides is equal to $(2n - 4)$ right angles, and that the sum of its exterior angles equals 4 right angles. They were able to construct figures of a given area, and to use simple geometrical algebra, for example to solve equations such as $a(a - x) = x^2$ by geometrical means.

The Pythagoreans also established the foundations of number theory, with their investigations of triangular, square and also perfect numbers (numbers that are the sum of their divisors). They discovered several new properties of square numbers, such as that the square of a number n is equal to the sum of the first n odd numbers (e.g. $4^2 = 16 = 1 + 3 + 5 + 7$). They also discovered at least the first pair of amicable numbers, 220 and 284 (amicable numbers are pairs of numbers for which the sum of the divisors of one number equals the other number, e.g. the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220).

Pythagoras is also credited with the discovery that the intervals between harmonious musical notes always have whole number ratios. For instance, playing half a length of a guitar string gives the same note as the open string, but an octave higher; a third of a length gives a different but harmonious note; etc. Non-whole number ratios, on the other hand, tend to give dissonant sounds. In this way, Pythagoras described the first four overtones which create the common intervals which have become the primary building blocks of musical harmony: the octave (1:1), the perfect fifth (3:2), the perfect fourth (4:3) and the major third (5:4). The oldest way of tuning the 12-note chromatic scale is known as Pythagorean tuning, and it is based on a stack of perfect fifths, each tuned in the ratio 3:2.



Pythagoras is credited with the discovery of the ratios between harmonious musical tones

The mystical Pythagoras was so excited by this discovery that he became convinced that the whole universe was based on numbers, and that the planets and stars moved according to mathematical equations, which corresponded to musical notes, and thus produced a kind of symphony, the "Musical Universalis" or "Music of the Spheres".

Short in Text Question

1. Explain the Contribution of Euclid.
   ………………………………………………………………………………………
   ………………………………………………………………………………………
2. Explain the Contribution of Pythagoras
   ………………………………………………………………………………………
   ………………………………………………………………………………………

1.5.5 Summary

In this lesson we discussed about the contribution of Euclid cut Pythagoras in mathematic. Euclid Greek mathematic of antiquity known primarily for the highly influential treatise on geometry the element.

1.5.6 Self Check exercise

Fill in the blanks:

1. Father of Geometry……………………………
2. The Pythagorean also established the formative of………………………

   1) Euclid 2) Number theory.

1.5.7 Suggested Questions:

1. Discuss the life sketch of Euclid.
2. Explain the life history of Pythagoras.
3. What are the main contributions of Euclid's and Pythagoras's in mathematics?

1.5.8 Suggested books:

1. Mangal,S.K.: "Teaching of Mathematics",New Delhi:Arya Book Depot,2008….

2.Sidhu,K.S.: "Teaching of Mathematics",New Delhi sterlimg Publication,1990

3. Mangal,S.K.: "Teaching of Mathematics",Ludhiana Parkash Book Depot

Mandatory Student Feedback Form

https://forms.gle/KS5CLhvpwrpgjwN98

Note: Students, kindly click this google form link, and fill this feedback form once.